Even and Odd Numbers

G'Day!

This is your math friend James. Today I am answering a question from Jack.

What are even and odd numbers good for?

What an interesting question! I wonder if Jack read issue 31, which talked about even and odd numbers a little bit.

Remember, a count of objects is even if we can group that many objects into pairs with none left over. A count is odd if there is always one object left over when you try this.
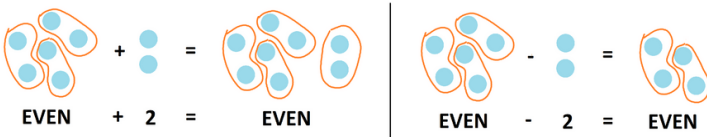
6 is even | **7 is odd**

People say that the number zero is even. Even though you can't make any pairs with zero objects, you absolutely don't have one object left over. Zero just can't be odd. Do you agree that that then means zero is even?

Adding two to an even number keeps it even. Subtracting two from it keeps it even as well.
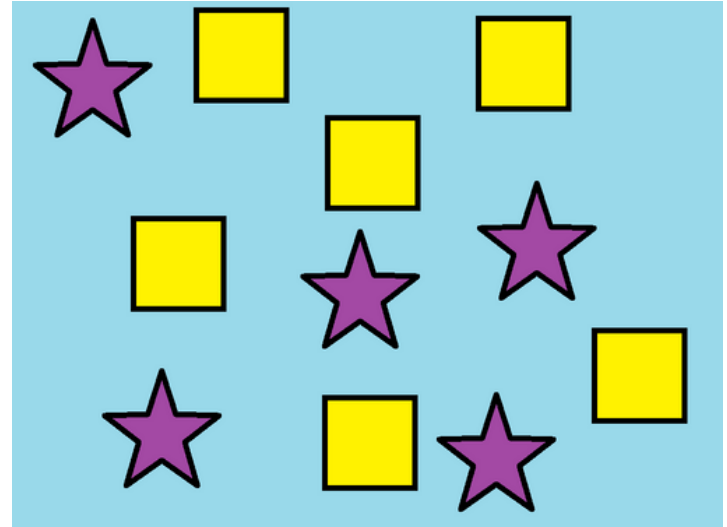
EVEN + 2 = EVEN | EVEN - 2 = EVEN

Adding two to an odd number keeps it odd. Subtracting two from it keeps it odd as well.

Can you draw some pictures to show this?

Now I am ready to give my answer to Jack's question.

What are even and odd numbers good for?
They are good for creating games you are sure to win!

THE STARS AND SQUARES GAME

Copy the picture above of 6 squares and 5 stars. Draw the shapes in pencil.

Your friend goes first. She erases two shapes from the page and then replaces them by drawing in one shape. But the single shape she draws has to follow this rule:

If she erases two shapes that are the same, she must draw a square.
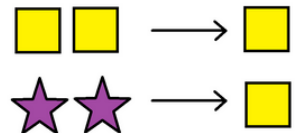
If she erases two shapes that are different, she must draw a star.
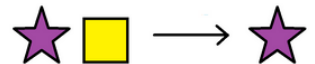
There is now one less shape on the page.

On your turn, you do the same: erase two shapes and replace them with one shape following the same rules.

You keep taking turns until there is one single shape left on the page.

If that single shape is a star, YOU WIN!
If it is a square, your friend wins.

The trick is that we started this game with an odd number of stars.

Each move either removes two stars or keeps the number of stars the same. (Think through this: What happens to the number of stars if we erase two squares and then draw the required shape? If we erase two stars? If we erase a star and a square?)

So, as we play the game, we'll never be able to eliminate all the stars: the count of them will forever stay odd.

This means, when we get to the point of the game where there is just one shape left, because we can't eliminate all the stars, that one shape must be a star. You simply must win!
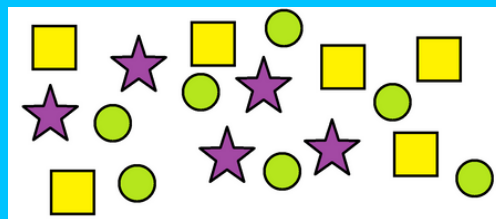
puzzle #1

If you play the stars-and-squares game starting with 5 stars and 1000 squares, are stars still sure to win?

If you play the game starting with 201 stars and 201 squares, who will win?

if you play the game with 10 stars and 10 squares, will the same person win over and over again?

puzzle #2

Here's another game. Start with 5 stars, 6 squares, and 7 circles on a page.

Each turn consists of erasing two different shapes and drawing in their place the third shape. (So, for example, if you erase a square and a star, you must draw a circle.) Your goal is to get down to a single shape. Can you?
(And how do I know that if you do succeed you'll be left with a square?)

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The NMF Weekly is written by mathematician Dr. James Tanton as a resource for friends and fans of the 2021 National Math Festival.

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