Dividing by Zero

G'Day!

This is your math friend James. Today I am answering a question from Venkat.

I want to know why division by zero gives infinity.

This is a tremendous question! The number zero has befuddled humankind for thousands of years and many people wonder, "Why is dividing by zero forbidden?" Venkat is thinking that maybe dividing by zero is possible, and that it gives the answer "infinity," whatever that actually means.

Let's see if I can help make sense of things. (I am not always good at this, so let me know if I am helping today!)

Let me first talk about division. What does it mean to divide things?

I personally like to think of division as sharing. For example, if I have 6 pies to share equally among 3 students, that ends up giving 2 pies per student.
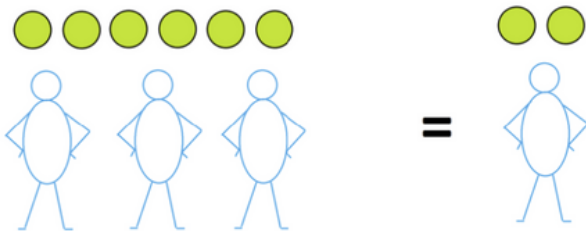

We write this as $6 \div 3 = 2$.

In the same way, $20 \div 5 = 4$, as sharing twenty pies equally among five students gives four pies per student. (Care to draw a picture of this?)

And $3000 \div 600 = 5$ since sharing 3000 pies equally among 600 students gives 5 pies per student. (Don't draw a picture of this!)

Let's now get quirky and ask: What's $1 \div 1/2$? That is, what's one divided by a half?

Whoa! Now "$1 \div (1/2)$" says that we are sharing one whole pie with half a student. Hmmm. Can we draw a picture of this?

$$1 \div (1/2) = 2$$
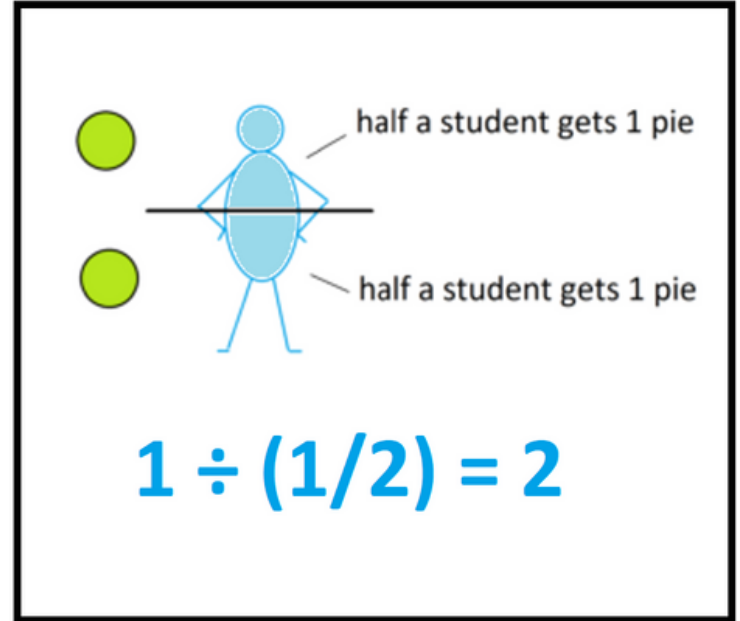
Yes we can!

The picture above shows what is going on. If we give 1 pie to half a student, then we are giving 2 pies to the whole student! Why? The lower half of the student gets 1 pie and the upper half of the student gets 1 pie.

This is fun!

What's $1 \div 1/3$? That is, what's one divided by a third?

Each third of a student gets 1 pie, so the whole student gets 3 whole pies. (Can you draw a picture for this?)

What's $1 \div 1/20$? That is, what's one divided by a twentieth?

Each twentieth of a student gets 1 pie, so the whole student gets 20 whole pies. (Can you draw a picture for this too?)

What's $1 \div 1/1000$? That is, what's one divided by a thousandth?

Each thousandth of a student gets 1 pie, so the whole student gets 1000 whole pies.

Can you guess where this is going?

Giving 1 pie to smaller and smaller fractions of a student gives more and more pies to the whole student.

I am not sure if I am really answering Venkat's question. I am not actually dividing by zero and I am not actually getting "infinity" in any of my answers, but it seems I am heading towards both of these!

puzzle #1

a) What's 4 ÷ (1/2)?

That is, if I give four pies to each half of a student, how much pie does a whole student receive?

b) What's 10 ÷ (2/3)?

That is, if two-thirds of a student receives ten pies, how much pie does a whole student receive?

Check out **MATHICAL** for award-winning math books for middle-schoolers and teens, the YouTube channel **NUMBERPHILE** for math videos galore, and **MORE MATH!** for even more resources. Wowza!

Here's a video from NumberPhile Problems with Zero

Do you have a math question for me to answer, or try to answer?

Write to me **at the website**.
Each week I'll pick a new question and give my thoughts on it!

But there is a more fundamental question about zero. Is "zero" even a number?

For example, numbers are meant to count things: "I have 2 thumbs" and "My house has 12 windows" and "The word "word" is a 4-letter word."

What does zero count?

If I say that that there are--right now--zero purple parrots sitting on my head is that because I actually counted zero purple parrots on my head? Or did I just observe that there are no parrots on my head?

For many thousands of years people did not think that zero was a number: you just don't count anything when you say you have zero of something! It wasn't until the 7th-century that Brahmagupta, a brilliant Indian scholar, said that even if we are not sure whether or not zero is a number, we can still do arithmetic with it as though it really is number. He showed the world how arithmetic with zero works.

But here's the thing: Even Brahmagupta said that dividing by zero should not be allowed!

puzzle #2

Two-and-a-half pies are shared equally among four-and-a-half students. How much pie does one whole student receive?

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The NMF Weekly is written by mathematician Dr. James Tanton as a resource for friends and fans of the 2021 National Math Festival.

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