## the nmf weekly

## Adding Two Odd Primes

G'Day!
This is your math friend James. Today I am answering a question from Clara.

## Why isn't the Goldbach Conjecture solved? <br> Doesn't adding two odd prime numbers give an even number?

This is asking a big question about a big topic. There's a lot to explain just to understand her fabulous question, yet alone answer it. (Clara has clearly been reading a lot about mathematics! Good for her!)

First, let me identify the words in this question that seem strange, tricky, or scary, or just important. Here are the ones I think that are.

| Goldbach | Solved | Even | Adding |
| :--- | :--- | :--- | :--- |
| Conjecture | Prime | Odd |  |

Can we make sense of these? Let's start with the ones I feel okay about.
[By the way: This is how I usually approach written problems I don't really understand. I list the specific words or ideas mentioned that seem important, and then try to remember or figure out or research what they each mean. I start with the easier ones first so that I can feel good and confident about making some progress!]

Even/Odd: Here's how I like to think about this. A count of objects is even if you can group the objects into pairs with none left over. For example, the picture above shows that 6 is even. A count of objects is odd if making pairs always leaves one left over. For example, 7 is odd.

Adding to two odd numbers: Can you see that if we take a picture of an odd number and combine it with a picture of another odd number, the leftover objects combine to make another pair? ODD + ODD = EVEN



6 is even


7 is odd
puzzle \#1

1. Is the sum of an even number and even number odd or even?
2. Is the sum of an odd number and an even number and an even number and an even number and an odd number and an odd number and an odd number even or odd?
3. Suzzy says that ten pages have been torn out of her textbook. (This is not good!) Arjun says: "The sum of the missing page numbers is odd." How does he know this?

We saw Issue 26 what a prime number is.
Prime Number: A number is prime if you cannot make any "meaningful rectangles" with that many objects. For example, with 12 dots we can make a 3 -by-4 rectangle and a 2 -by- rectangle. With 13 dots we can only make a single row or a single column of dots. (Try it!). The number 13 is prime.

The list of prime numbers starts $2,3,5,7,11,13$, 17, 19, 23, 29, 31, 37, 41, 43, 47, ....

Okay ... The extra-scary words next.

Goldbach: If you type these letters into the internet you see that Christian Goldbach was a German mathematician who lived about 300 years ago, and that he seems to be famous for his "conjecture."

Now we have to figure out that word is!
Conjecture: If you look up the definition of this word, we learn it is a statement that someone thinks might be true, but doesn't know if it really is true.

Okay. So Christian Goldbach said something that he thought might be true, but didn't know if it was true. What was it? The internet helps again!

The Goldbach Conjecture: Goldbach noticed that every even number 6 and bigger seems to be the sum of two odd prime numbers. We wondered if this is always true.

$$
\begin{array}{rlrlrl}
6 & =3+3 & 8 & =3+5 & & 10=3+7 \\
12 & =5+7 & 14 & =7+7 & & 16=5+11 \\
18 & =5+13 & & 20 & =7+13 & \\
\text { etc. } .
\end{array}
$$

Other people started wondering too. For 300 years now, mathematicians have been wondering if Goldbach is right. No one on this planet knows!

Check out MATHICAL for award-winning math books for middle-schoolers and teens, the YouTube channel NUMBERPHILE for math videos galore, and MORE MATH! for even more resources. Wowza!

Here's a video from NumberPhile about the Goldbach Conjecture.

## Do you have a math question for me to answer, or try to answer?

Write to me at the website.
Each week l'll pick a new question and give my thoughts on it!

Computers have checked every even number up to 4,000,000,000,000,000,000, and it holds every time. But what about 4,000,000,000,000,000,002 and $4,000,000,000,000,000,004$ and...$?$

Now to Clara. She is absolutely right that adding together two odd prime numbers does give an even number. But Goldbach was wondering about the other direction of thinking: Is every even number the sum of two odd prime numbers?

That direction of the question is mighty tricky remains one of the most famous unsolved questions in all of mathematics.

The numbers that come from rectangle arrangements, with one side of the
rectangle one unit longer than the other, are called the "oblong numbers."

|  |  | 0000 | 00000 |
| :---: | :---: | :---: | :---: |
|  | 000 | 0000 | 00000 |
| 00 | 000 | 0000 | 00000 |
| $1 \times 2=2$ | $2 \times 3=6$ | $3 \times 4=12$ | $4 \times 5=20$ |

Take the number 41, which is prime.
If you add the oblong numbers to it, do you keep getting prime number answers?

$$
\begin{aligned}
& 41+2=43 \text { prime } \\
& 41+6=47 \text { prime } \\
& 41+12=53 \text { prime } \\
& 41+20=61 \text { prime } \\
& \text { etc. }
\end{aligned}
$$

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The NMF Weekly is written by mathematician Dr. James Tanton as a resource for friends and fans of the 2021 National Math Festival.

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