

SQUARES: Solutions

puzzle #1

In a five-by-five grid ...

There are **25** one-by-one squares.

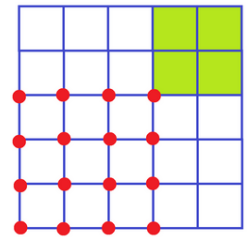
Show that there **16** two-by-two squares, **9** three-by-three squares, and **4** four-by-four squares.

There is **1** big five-by-five square.

What do you notice about the numbers 25, 16, 9, 4, and 1? Can you explain why what you are noticing must be true?

ANSWER TO PUZZLE 1:

To count the two-by-two squares, for example, ask: *Where could the bottom left corner of a two-by-two square be?* We see that we get a four-times-four array of possible bottom left corners. The count of two-by-two squares is thus the square number 16.



In this way we see that the count of squares of any particular size must be a square number!

puzzle #2

Draw a tilted square of area 8.

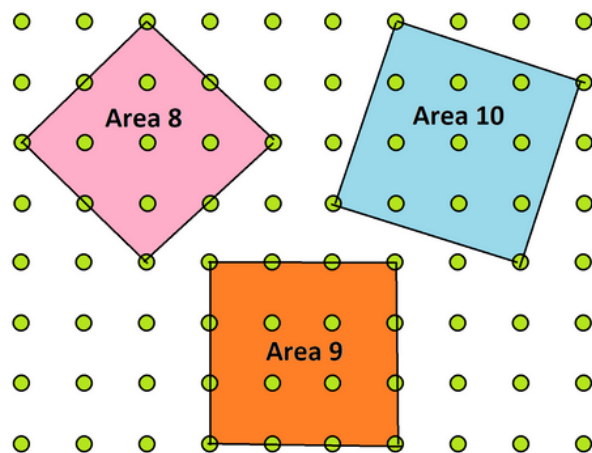
Draw a non-tilted square of area 9.

Draw a tilted square of area 10.

For which of the numbers 1 through 20 is it possible to draw a square of that area?

ANSWER TO PUZZLE 2:

It is possible to draw squares of areas 1, 2, 4, 5, 8, 9, 10, 13, 16, 17, 18, and 20. (Hard question: Why are the other areas impossible to produce?)



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The NMF Weekly is written by mathematician Dr. James Tanton as a resource for friends and fans of the 2021 National Math Festival.



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