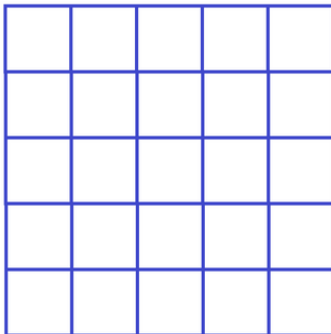## Counting Squares

G'Day!

This is your math friend James. Today I am answering a question from Lexie.

How do you count squares?

This is a curious question. I bet Lexie is thinking of a puzzle about counting squares and is finding it tricky. And she is right! It can be tricky to count squares.

For example, here's a square-counting puzzle:

How many squares are in this picture?

Most people first say that the answer is 25.
But puzzle specialists say that the answer is 55!

How is that possible?
Do you see fifty-five squares in the picture?

There are certainly 25 little one-by-one squares. But there are also some two-by-two squares, and some three-by-three squares, and some four-by-four squares, and there's 1 big five-by-five square.

Okay, there are a lot of squares!
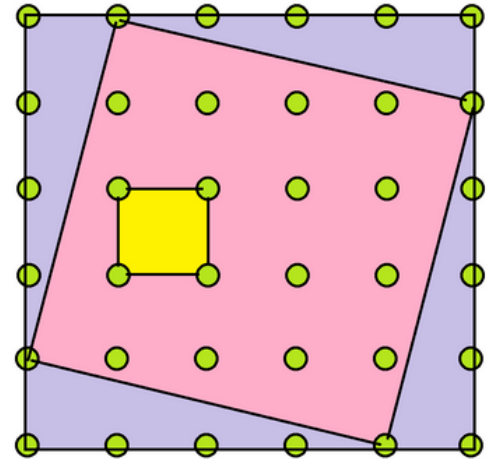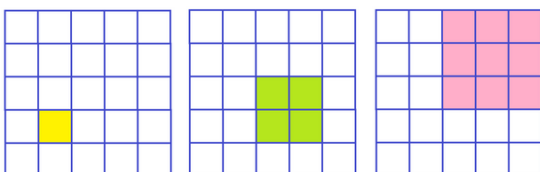
puzzle #1

In the five-by-five grid …

There are **25** one-by-one squares.

Show that there **16** two-by-two squares.
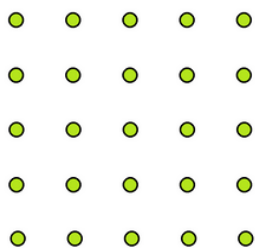
Show that there are **9** three-by-three squares.

Show that there are **4** four-by-four squares.

And there is **1** big five-by-five square.

This makes 25+16+9+4+1 = 55 squares.

But what do you notice about the numbers 25, 16, 9, 4, and 1? Can you explain why what you are noticing must be true?

Or perhaps Lexie is counting squares in a picture like this with each corner of a square lying on a dot?

We can certainly find one-by-one squares like before, and two-by-two squares, and so on. But now it is also possible to find tilted squares!
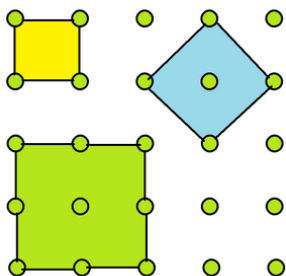
Can you find 50 squares in total?

Do you have a math question for me to answer, or try to answer?
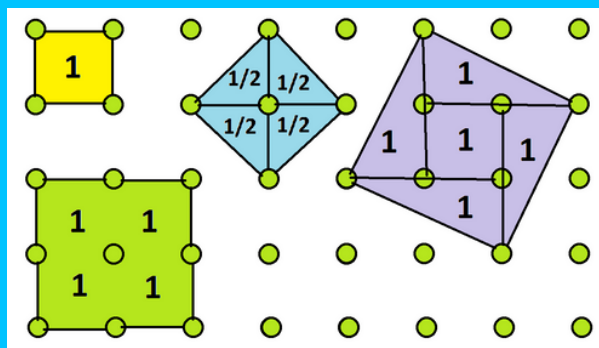
Write to me at the website. Each week I'll pick a new question and give my thoughts on it!

puzzle #2

Let's keep playing with squares drawn on grids of dots.

Suppose the yellow square has area 1.

Then the green square has area 4 and the blue square has area 2.

Can you see why the purple square has area 5? (View the triangles you see as half rectangles.)

Draw a tilted square of area 8.
Draw a non-tilted square of area 9.
Draw a tilted square of area 10.

For which of the numbers 1 through 20 is it possible to draw a square of that area?

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The NMF Weekly is written by mathematician Dr. James Tanton as a resource for friends and fans of the 2021 National Math Festival.

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