

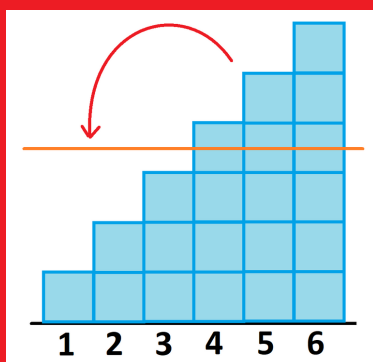
Average: SOLUTIONS

puzzle #1

This picture shows that the average of the first 6 counting numbers is three-and-a-half. (Do you see this?)

What is the average of the first 10 counting numbers?

What is the average of the first 100 counting numbers? The first 1000 counting numbers?



ANSWER TO PUZZLE 1:

We can "level out" 1 and 10 to $5\frac{1}{2}$, 2 and 9 to $5\frac{1}{2}$, 3 and 8 to $5\frac{1}{2}$, 4 and 7 to $5\frac{1}{2}$, and 5 and 6 to $5\frac{1}{2}$, and so all the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 level out to $5\frac{1}{2}$.

The numbers 1, 2, ..., 100 level out to $50\frac{1}{2}$.

The numbers 1, 2, ..., 1000 level out to $500\frac{1}{2}$.

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The NMF Weekly is written by mathematician Dr. James Tanton as a resource for friends and fans of the 2021 National Math Festival.

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puzzle #2

Twenty students are in a room. Their ages are such that each person can point to ten other people in the room and say

"My age is the average of their ten ages."

Bjorn is in the room and he is nine years old. Talathia is also in the room. How old is Talathia?

ANSWER TO PUZZLE 2:

Think of the youngest person in the room. If their age is the average of ten other people in the room, then all those ten people must have that same youngest age. (Imagine the "leveling out" picture. If the final level equals the shortest stack, then all stacks must have been that same shortest height.)

So there are 11 people in the room with the same youngest age.

By the same reasoning, there are 11 people in the room the same oldest age.

As there are only 20 people in the room in total, this youngest age and oldest age must be the same. This means that Talathia must be 9 years old too (as you would have guessed the answer had to be!)

