

On Average

G'Day!

This is your math friend James. Today I am answering a question from Luc.

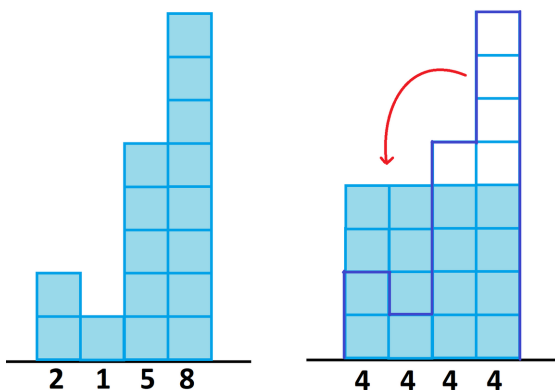
What is average?

This is a wonderful and curious question as one has to think a little bit about what Luc might mean by it. Maybe the question is: What does the word "average" mean? Or maybe Luc knows what it means and wants to know some examples of things that might be considered average. I think my job is to write about both of those ideas.

So, what does "average" mean?

Here are four numbers: 2, 1, 5, 8. I am going to think of them four as stacks of blocks.

Now, suppose we try to level out the stacks. We can take some blocks from the tall stack and place them on the short stack, and keep doing this until we have four level stacks of 4 blocks each. (Can you see what I am doing in my picture?)



We had a total of $2+1+5+8 = 16$ blocks in four stacks, and leveling them out gives us $16 \div 4 = 4$ blocks per stack. People say that 4 is the **average** of 2, 1, 5, and 8. It's the "level" number in my picture.



If we had stacks of 9, 2, and 4 blocks, that's $9+2+4 = 15$ blocks in total, and we level out the three stacks, we'd see $15 \div 3 = 5$ blocks per stack. (Draw a picture of this and see if I am right.) The average of 9, 2, and 4 is

$$(9+2+4) \div 3 = 5.$$

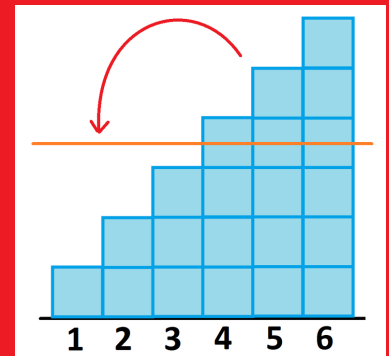
To find the average of any set of numbers, just add them up and divide the sum by how many numbers you have. That's what our leveling pictures are doing.

puzzle #1

This picture shows that the average of the first 6 counting numbers is three-and-a-half. (Do you see this?)

What is the average of the first 10 counting numbers?

What is the average of the first 100 counting numbers? The first 1000 counting numbers?



The average gives a sense of the usual count in a scenario. For example, if over four days I baked 2 cakes, then 1 cake, then 5 cakes, and then 8 cakes, that would have been the same as me baking 4 cakes each day.

Or, if I got a score of 9 out of ten on my first test, 2 out of ten on my second test, and 4 out of ten on my third test, that would be the same as me having scored 5 out of ten on each test.

I just looked up the "average age of everyone in the world" and the website said 28.1 years. (I wonder if this is true. How would we know?)

If this number is true, it means that if we added up all the ages of everyone on this planet, we'd get the same sum as if we assumed that everyone is 28.1 years old. Of course there are some people who are 0 years old and some people who are very old (who's the oldest person right now?), but if we "level out" everyone's ages, we'd get the number 28.1.

Check out [MATHICAL](#) for award-winning math books for middle-schoolers and teens, the YouTube channel [NUMBERPHILE](#) for math videos galore, and [MORE MATH!](#) for even more resources. Wowza!

Here's a video about making use of average behavior in [ROCK, PAPER, SCISSORS](#).

Do you have a math question for me to answer, or try to answer?

Write to me [at the website](#). Each week I'll pick a new question and give my thoughts on it!

Sometimes you can get some funny average results. For example, another website I saw says that, on average, each household in America has 1.6 pets. Of course, you can't have 1.6 pets, but this number tells us that, typically, each household has 1 or 2 pets, slightly more having 2 instead of 1. (Again, of course, there are many households with 0 pets and many with many pets. So, averages only give you a feel for what is typical.)

My favorite joke average is the following: The squares on a chess board are either black or white. So, on average, each square of a chessboard is grey.

We have to be careful when trying to make sense of what an average means!

puzzle #2

Twenty students are in a room. Their ages are such that each person can point to ten other people in the room and say

"My age is the average of their ten ages."

Bjorn is in the room and he is nine years old. Talathia is also in the room.

How old is Talathia?

About the Author: Dr. James Tanton

The NMF Weekly is written by mathematician Dr. James Tanton as a resource for friends and fans of the 2021 National Math Festival.

Learn more at globalmathproject.org/nmf-weekly & nationalmathfestival.org

