

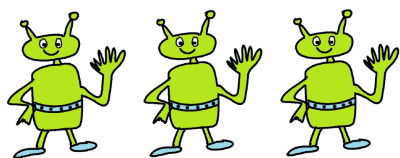
## Infinity

G'Day! This is your math friend James. Today I am answering a question from Milli.

### What is infinity?

What a super question! Is infinity a number? A really big number? A number so big that it is bigger than all other numbers? (Is such a number possible?)

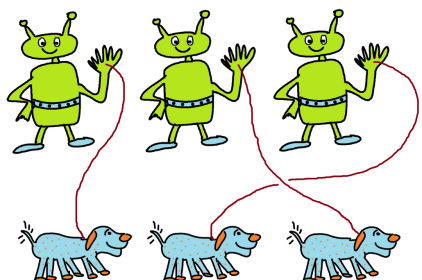
When we first think of numbers, we usually think of words that are used to describe collections of things. For example, here is a collection of Martians. We associate the word "three" with this collection.



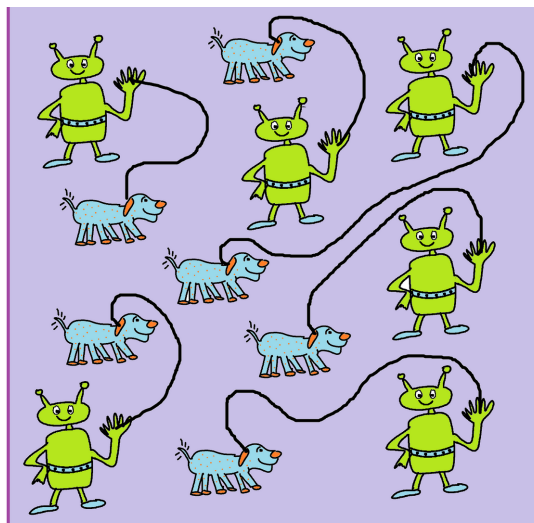
And here is a collection of space dogs. We associate the word "three" with this collection too.



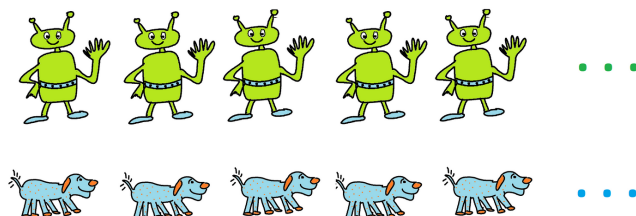
And without even thinking or saying the word "three" we can see that these two collections are the same size by drawing leashes that connect each dog to one Martian and each Martian to one dog.



Look at the picture at the top right. Without even counting, the leashes tell us that there are just as many Martians as dogs in that picture. The two collections are the same size. (I love the fun--and power!--of just drawing leashes!)

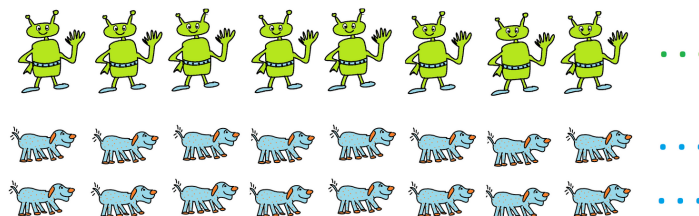


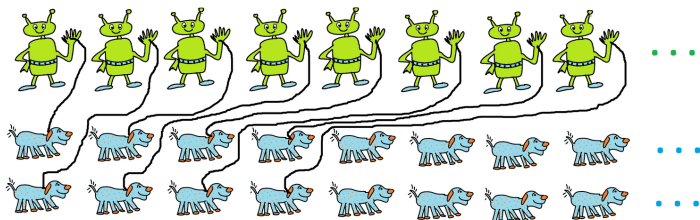
Now look at this next picture. It's meant to be a line of Martians that goes on forever to the right. Below them I have a line of dogs that goes on forever to the right too. And I bet you can imagine in your mind drawing leashes that successfully connect Martians to dogs and vice versa.



And even though I haven't answered Milli's question, we would all probably agree I have drawn two infinite sets, and that the leashes show that these two sets are the same size.

But crazy things can occur when comparing infinite collections. For example, look at the picture below. It looks like the number of space dogs is twice as infinite as the number Martians. Is it? The picture on the next page says no!



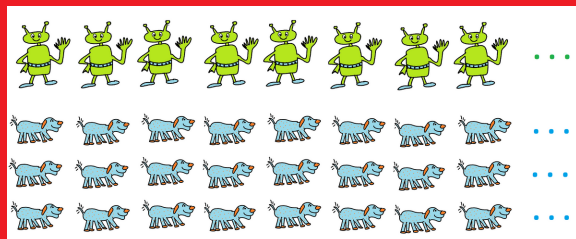


Do you see that each and every Martian holds a leash to a dog, and that each and every dog is leashed to one Martian?

These leashes show that this "double infinity" of space dogs is just as infinite as the single infinity of Martians. Whoa!

## puzzle #1

Can you show that a "triple infinity" is just as infinite as a single infinity?



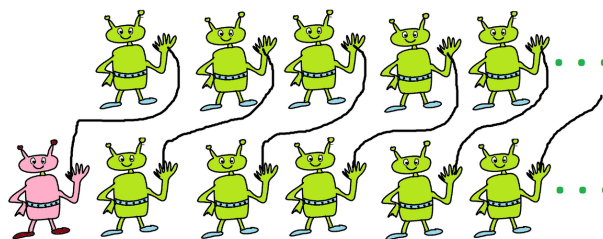
Check out [MATHICAL](#) for award-winning math books for middle-schoolers and teens, the YouTube channel [NUMBERPHILE](#) for math videos galore, and [MORE MATH!](#) for even more resources. Wowza!

Have a look at Richard Evan Schwarz's prize-winning book [REALLY BIG NUMBERS](#).

Do you have a math question for me to answer, or try to answer?

Write to me [at the website](#). Each week I'll pick a new question and give my thoughts on it!

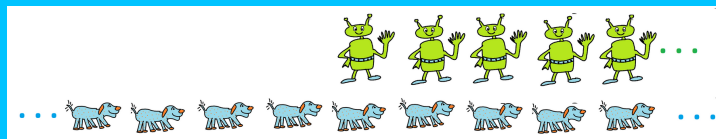
This picture shows an infinite set of Martians and a second infinite set of Martians, but with an extra Martian added to the collection. Leashes show that these two collections are still the same size!



Okay. Here's my answer to Milli's question. Mathematicians call a collection of objects **infinite** if, when you draw a second copy of the collection but add one more object to it, you can successfully draw leashes between the original collection and the "slightly bigger" copy. Infinite sets are the ones that behave this weird way!

## puzzle #2

This picture shows a line of Martians going off infinitely far to the right and a line of space dogs going infinitely far both to the right and to the left! Is it possible to leash each Martian to a dog and each dog to a Martian? Or is this "double infinity" of dogs a new type of infinity?



### About the Author: Dr. James Tanton

The NMF Weekly is written by mathematician Dr. James Tanton as a resource for friends and fans of the 2021 National Math Festival.

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