

Is 1 prime?

G'Day! This is your math friend James. Today I am answering a question from Simon.

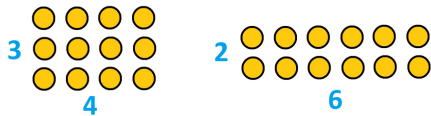
Is one a prime number?

This is a great question, and one that confuses many people. Let me see if I can sort out what the issues are.

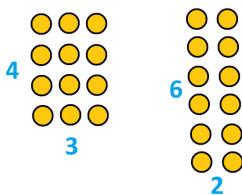
But let's start by having some fun with dots!

Question: **How many different rectangles can we make with 12 dots?**

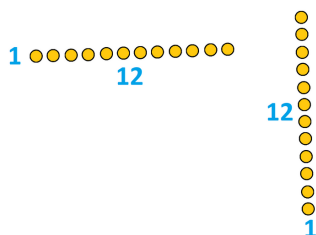
Well, we can certainly make a 3-by-4 rectangle and a 2-by-6 rectangle.



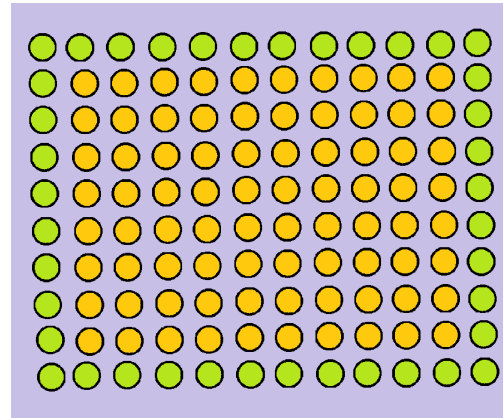
We can also make a 4-by-3 rectangle and a 6-by-2 rectangle. (But some people might not consider these to be new rectangles. Why?)



And there are two more rectangles that we can make: the 1-by-12 and 12-by-1 rectangles. But some people consider these rectangles "obnoxious" and not really rectangles. (What do you think?)

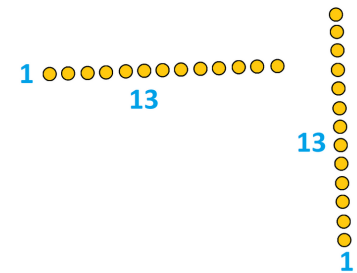


With 12 dots we can make four "real" rectangles and two obnoxious rectangles.



Let's add an extra dot to our play:

How many different rectangles can we make with 13 dots?



We can only make obnoxious rectangles, two of them!

And this is what people noticed throughout history. Some counts of dots, like 12, are nice and allow us to make nice rectangles along with the two obnoxious rectangles; and some numbers, like 13, are obnoxious and allow us to make only the two obnoxious rectangles.

puzzle #1

How many different rectangles can one make with 100 dots?

How many different rectangles can one make with 101 dots?

People call the obnoxious numbers, the counts of dots that make only the two obnoxious rectangles, **prime numbers**. The counts of dots that make at least one nice rectangle are called **composite numbers**.

Here are the first few prime numbers:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41.

Here are the first few composite numbers:

4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22 .

Do you see that the number 1 is missing from both lists?

How many different rectangles can we make with just 1 dot?

1 ●
1

One! Just the 1-by-1 rectangle.

This is why people are confused by the number 1. Is this really a rectangle?

Check out **MATHICAL** for award-winning math books for middle-schoolers and teens, the YouTube channel **NUMBERPHILE** for math videos galore, and **MORE MATH!** for even more resources. Wowza!

James Grime of NUMBERPHILE made a video to explain too Why 1 is Not a Prime Number.

Do you have a math question for me to answer, or try to answer?

Write to me at the website. Each week I'll pick a new question and give my thoughts on it!

With each number different from 1 we can make, at the very least, two rectangles: the two obnoxious rectangles. We can often make more rectangles.

To answer Simon's question:

A number is **composite** if you can make more than two rectangles with that many dots.

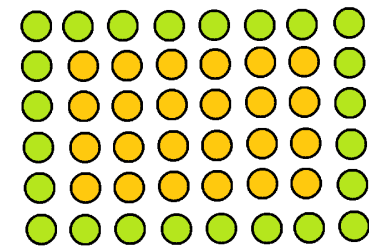
A number is **prime** if you can only make two rectangles with that many dots.

The number 1 is neither prime nor composite. You can only make one rectangle with that many dots (and it is not much of a rectangle!).

puzzle #2

This picture has 24 yellow dots in a 4-by-6 rectangle surrounded by a single layer of green dots. Count the green dots. There are also 24 of them. WHOA!

Create another picture like this. Make a rectangle of a different size with yellow dots



and a single-layer border of green dots so that the count of green and yellow dots are again the same.

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The NMF Weekly is written by mathematician Dr. James Tanton as a resource for friends and fans of the 2021 National Math Festival.

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