Showing Your Work

G'Day! This is your math friend James. Today I am answering a question from Ollie.

Why is it important to show your work in math?

This is a fascinating question! I think the answer to it all depends on what you are trying to say to whoever is going to read what you write.

For instance, I am not in school any more and so when I am writing math and writing about thinking about math, I have to check each time whether or not I am communicating what I want to communicate.

For example, if I write

$$35 \times 6 = 210$$

then I am communicating to you just a math fact, nothing more. You don't know how I got the answer. But maybe the answer is all you need to know from me.

But suppose I wrote instead

35 x 6 equals 7 x 5 x 2 x 3.

The 2x5 part is easy, that's 10.
The 7x3 part is fine too, it's 21.

So 35x6 is the same as 21 x 10.
And that I can do. It's 210.

Then I am conveying to you so much more! I am showing you that if you can see a 2 and 5 hidden in a multiplication problem, then you can use it to your advantage! Multiplying by 10 is easy to do.

So, showing my work here was about communicating to you a math idea.

Can you work out each of 15 x 18 and 5 x 24 and 12 x 25 this way?

It's all about communicating a thinking idea that helps the reader.

But every now and then you see on social media a "math problem that stumps the internet." It is usually something someone wrote with the deliberate intention to confuse the reader, not to help them understand.

Here's a recent math problem that was posted.

Compute $8 \div 2(2+2)$.

One is taught in school to evaluate whatever is inside the parentheses first, and doing so gives

$$8 \div 2(4).$$

Everyone agreed with that.

But then things got confusing. Do you do the multiplication first (which still have parentheses present) and the division second, or do you now just go "left to right" and read this as "8 divided by 2, and then times 4"?

The first way gives $8 \div 8$ which leads to an answer 1, and the second way gives the answer 16. (Do you see this?)

So, which is the correct answer?

The usual rule is that if there are nothing but multiplications and divisions in an arithmetic problem, do them left to right. In which case, we have eight divided by two is 4, and multiplying by four then gives 16. This the correct school answer.

But my point in sharing this example is the following: If there is possible confusion in what you write it is YOUR responsibility as the writer to rewrite what you have to make it clear and simple for the reader.

The author of the internet problem could have written (8÷2)(2+2) to help everyone get 16.

"Showing your work" is about being a good communicator and making it easy and clear for people to understand what you say and why you say it.

But for Ollie's question.

Who reads a student's work? The teacher.
So, who is a student writing for? Just the teacher!

I think teachers are working to help their students be better communicators. And that is a mighty good thing, a skill that is important well beyond just math!

Check out **MATHICAL** for award-winning math books for middle-schoolers and teens, the YouTube channel **NUMBERPHILE** for math videos galore, and **MORE MATH!** for even more resources. Wowza!

Actually, do check out Mathical for examples of communicating mathematics beautifully!

Do you have a math question for me to answer, or try to answer?

Write to me at the website.
Each week I'll pick a new question and give my thoughts on it!

puzzle #1

Here are some more problems that have appeared on the internet. Do you see why they are confusing to people? What are the answers to them if you follow the rules from school?

Compute

a) $1 + 2 \times 3$

b) $6 \div 2(1+2)$

c) $15 - 1(12 \div 4 + 1)$

puzzle #2

Sometimes math puzzles are designed to seem that they are not communicating information well, but they actually are! Can answer this famous puzzle?

One day a woman decides to take a three-mile walk. She starts by heading directly south for one mile, trotting along at a happy pace, admiring the sunshine and the wildlife. Then she turns left and heads directly east for one mile, all the while enjoying the smell of the sweet air and the glorious sights of nature around her. Next, she turns left one more time, exactly , and heads directly north just for one more mile. Surprisingly, after this third one-mile stretch she finds herself back to where she started!

What color was the bear she saw on her walk?

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The NMF Weekly is written by mathematician Dr. James Tanton as a resource for friends and fans of the 2021 National Math Festival.

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