Subtraction

G'Day!

This is your math friend James. Today I am answering a question from Jonathan.

Why do you do subtraction weird?

I guess Jonathan has been watching some YouTube videos of mine like this one, "I don't believe subtraction exists" and perhaps this one too, "Fun with long subtraction." And it is true that I do like to think about standard ideas in mathematics in ways that can seem strange and weird. Jonathan is asking why I do that, and that is a good question!

The simple answer is that I don't like hard work, and I will work very hard to think about math and how to avoid hard work when doing math! For example, look at this subtraction problem.

$$\begin{array}{r} 1\,0\,0\,5 \\ -\ 8\,7\,6 \\ \hline = \end{array}$$

If you try it the standard way, it's hard. There are to many borrows! So my brain says: "There's got to be an easier way!"

So, that's what I do. I spend a lot of time to trying to think of easier ways to do things. And here's the weird thing: I tend to spend more time thinking about a problem than I would if I just did the problem the normal way! But that thinking is way more fun and insightful and, in the end, speed in mathematics is not important. (If I wanted the answer fast, I'd just get out my calculator!)

What then are some different and clever ways to think about this particular subtraction problem?

One approach could be to do the problem in an "upside down" kind of way. Start with the number 876 and ask how much we need to add to it to get to 1005.
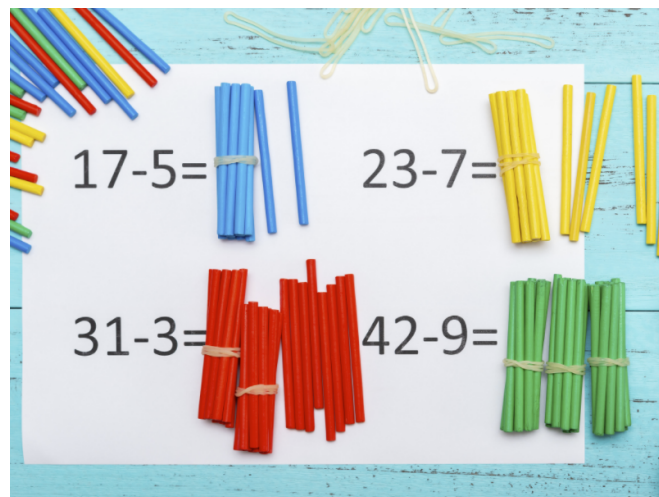
Adding 4 to 876 gets me to 880.
Adding 20 then gets me to 900.
Adding 100 then gets me to 1000.
And adding 5 more makes that 1005.

So, all in all, adding

$$4+20+100+5 = 124 + 5 = 129$$

gets me from 876 to 1005. The answer must be 129.

Another approach is to focus on what I didn't like about the problem and try to make that issue go away. For example, I thought there were "too many borrows" to do. Can I adjust the problem so that there no borrows at all? That would be wild!

puzzle #1

a) What's 20 take away 10?

b) I have to read pages 10 through 20 of my novel for homework tonight. How many pages will I actually read?

c) I am 20th in line for lunch and I see that my friend is ahead of me and is 10th in line. How many people are between us?

I am trying to work out the difference between

1005 and 876.

That is the same as the difference between

1004 and 875.

(Slide each number down by one on the number line!) That is the same as the difference between

1003 and 874,

which is the same as the difference between

1002 and 873,

and between 1001 and 872,

and between 1000 and 871,

and between 999 and 870.

Working out this last one requires no borrowing and I can just see the answer is going to be 129.

$$999 - 870 =$$

Whoa!

Another way to avoid borrows is to just not do them! What do you think of this answer?

$$1005 - 876 = 1 \; -8 \; -7 \; -1$$

Here I said:

5 take away six is -1.
0 take away 7 is -7.
0 take away 8 is -8.
1 take away nothing is 1.

And how might we unravel that answer?

Well, the 1 in the thousands place represents 1000,
the -8 in the hundreds place is really -800,
the -7 in the tens place is really -70,
and the -1 in the units place is -1.

So the answer is 1000 - 800 - 70 - 1.

This is two hundred take away seventy, giving me 130, take away one more, to give 129. There it is again! How fun!

Do you have a math question for me to answer, or try to answer?

Write to me **at the website.**
Each week I'll pick a new question and give my thoughts on it!

puzzle #2

What am I doing in this picture to get the answer 129 yet again?

$$1005 - 876 = {}^1 1 \; {}^1 0 \; {}^1 0 \; 5 - 1986$$
$$0129$$

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The NMF Weekly is written by mathematician Dr. James Tanton as a resource for friends and fans of the 2021 National Math Festival.

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