Proportional Reasoning

G'Day!

This is your math friend James. Today I am answering a question from Kaden.

What is "proportional reasoning"?

Ooh! I wonder what Kaden was reading or seeing or hearing to come across this scary-sounding phrase?

The phrase "proportional reasoning" is just the name used by textbook writers and teachers to quickly describe to each other some mathematics they are meant to teach. Students don't need to know these words.

But the math they represent is fun! It's all about using common sense. (That's the "reasoning" part of the name.) Let's do an example.

I grew up in Australia. Emus are very big birds that live in Australia and they have very big eggs. The picture shows an emu egg next to a chicken egg. (I happen to have an emu egg in my kitchen!)

I don't think anyone cooks with emu eggs (you can't buy a carton of them in a grocery store, for example), but suppose I am making breakfast for a very large group of people. I happen to know, with one egg, I can make 9 omelets.

1 egg   <---------->   9 omelets

That's not enough omelets. Common sense tells me that if I double the number of eggs, I'll double the number of omelets I can make.

2 eggs <-----------> 18 omelets

Still not enough!

If I now increase the number of eggs by a factor of ten, common sense tells me that I'll be able to make ten times as many omelets.

20 eggs <----------> 180 omelets.

That's too many omelets: I only need half as many as this. Well, common sense says to halve the number of omelets, just halve the number of eggs.

10 eggs   <--------> 90 omelets.

We are doing proportional reasoning!

If ever you are in a situation where you are comparing two different amounts or measurements, and common sense tells you if you double, or triple, or halve one quantity, the other one must double, or triple, or halve too, then you are doing proportional reasoning.

But don't worry about the words, just do the math! We're looking for two quantities that naturally scale up and down together in tandem.

puzzle #1

If it takes 4 minutes for me to read 3 pages of my book, how long would it take me to read 15 pages?

If it takes 22 minutes for one sock to dry on my clothesline, how long will it take for three socks to dry?

Proportional relationships appear in all sorts of places. If I am traveling at a constant speed of 30 miles per hour, then in 1 hour, I will travel 30 miles.

1 hour <--------> 30 miles

Common sense tells me if I triple the amount of time I drive I'll cover triple the number of miles.

3 hours <--------> 90 miles

If I want to cover only 9 miles, scale this by a factor of one tenth

3/10 hours <----> 9 miles

to see it will take me 3/10 of an hour (18 minutes) to do this.

Sometimes relationships can be "anti-proportional." (I am making up that term.) For example, if takes 6 people 4 hours to stuff a box of envelopes, presumable doubling the number of people will halve the amount of time needed to complete the job!

6 people <----> 4 hours
12 people <----> 2 hours

Here's a famous puzzle that combines proportional and anti-proportional reasoning together!

If it takes 6 cats, 2 days, to catch 25 rats, how long will it take 4 cats to catch 100 rats?

6 cats <---> 2 days <---> 25 rats
4 cats <---> ? days <---> 100 rats

Here comes some tricky thinking!

Let's keep the number of rats the same, but change the number of cats to 2, namely, one third of them. It will take these cats three times as long to catch 25 rats.

2 cats <---> 6 days <---> 25 rats

Let's now double the number of cats. The number of days they need will now halve.

4 cats <---> 3 days <---> 25 rats

Now we've got the right number of cats. If we quadruple the number of rats they need to catch, they will take four times as long.

4 cats <---> 12 days <---> 100 rats

We see it will take 4 cats 12 days to catch 100 rats!

Do you have a math question for me to answer, or try to answer?

Write to me at the website. Each week I'll pick a new question and give my thoughts on it!

puzzle #2

How many rats can eight cats catch in half a day?

About the Author: Dr. James Tanton

The NMF Weekly is written by mathematician Dr. James Tanton as a resource for friends and fans of the 2021 National Math Festival.

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