the nmf weekly

Ask your math friend, James

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What comes next?

G'Day!

This is your math friend James. Today I am answering a question from Maggie.

What number is next? 1 2 4 8 16 14 10 __

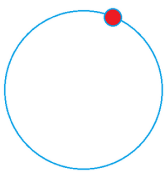
I will confess: I don't have a good guess as to what number should be next! Do you? (I hope Maggie will write to me and let me know what she is thinking.)

But I do know that patterns can be deceiving and that "what comes next" can always be a big surprise. My favorite example of this is the famous "Dots on a Circle" puzzle.

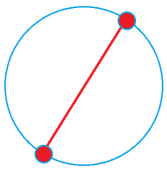
Draw some circles. On the first circle draw one dot. On the second circle draw two dots. On the third circle, draw three dots. And so on.

Then, with a ruler, connect each and every two dots in each circle with a straight line. This cuts the circle into pieces, like cutting a pizza.

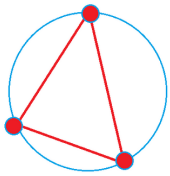
Count the number of pieces you get.



1 piece

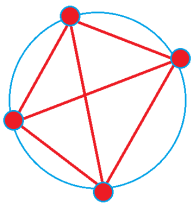


2 pieces

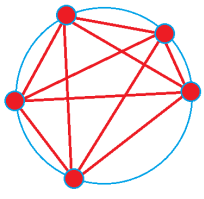


4 pieces

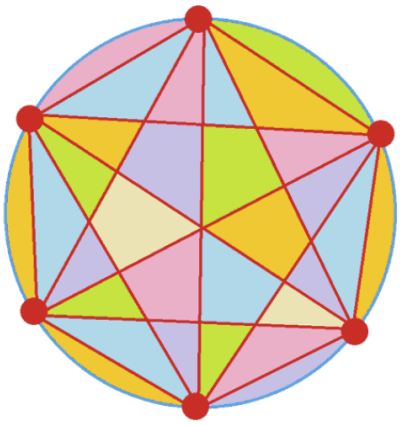
We first get 1, 2, and the 4 pieces for one, two, and three dots on the circle. For five dots, we then get 8 pieces. And for six dots, 16 pieces. (Check this for yourself!)



8 pieces



16 pieces



30 pieces

From 1, 2, 4, 8, and 16 pieces for one, two, three, four, and five dots on a circle, how many pieces would you then guess for six dots on a circle? I would personally guess 32. (Can you see why?)

Try it! You don't get 32 pieces!

This is why, as a mathematician, I don't trust patterns! I might be excited by a pattern, but I always want to make sure I have a logical reason for believing why a pattern is true.

puzzle #1

The picture on this page has six dots evenly spaced on the circle and we see the lines connecting the dots divide the circle into 30 pieces.

But what if we did NOT make arrange the dots symmetrically? Try arranging six dots on a circle in a different way and connecting all the pairs of dots. Can you get more than 30 pieces? (I bet you can!)

How many pieces can you get?

I do have an answer to Maggie's question, but it is complicated and crazy! And it is going to come out of the blue!

Do you know the following famous arithmetic triangle?

```
                1
              1   1
            1   2   1
          1   3   3   1
        1   4   6   4   1
      1   5   10  10  5   1
    1 6   15  20  15  6 1
  1 7   21  35  35  21 7 1
1 8  28 56   70   56 28 8 1
```

This triangle has 1s down the sides and each number in the interior is the sum of the two numbers just above it. For example, the 20 you see is from 10 + 10 just above it, and each of the 21s you see is 6 + 15 just above it. And so on.

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Do you have a math question for me to answer, or try to answer?

Write to me at the website.
Each week I'll pick a new question and give my thoughts on it!

This triangle of numbers has been discovered and studied by scholars from ancient China, and India, and the Persia, and more recently by western European scholars too.

Now look at just the first seven entries in each row. (I added some zeros so that each row has seven entries.)

```
                1 0 0  0 0 0 0
              1 1 0 0  0 0 0
            1 2 1 0  0 0 0
          1 3   3 1  0 0 0
        1 4   6   4 1  0 0
      1 5   10  10 5  1 0
    1 6   15  20  15 6 1
  1 7   21  35  35  21 7
1 8  28 56   70   56 28
```

In each row add together the first five numbers, and then subtract 17 times the sixth number, and then add 55 times the seventh number. For the first row you get

$$1 + 0 + 0 + 0 + 0 - 17x0 + 55x0 = 1.$$

Then you get 2, 4, 8, 16 and then

$$1 + 5 + 10 + 10 + 5 - 17x1 + 55x0 = 14$$

and then

$$1 + 6 + 15 + 20 + 15 - 17x6 + 55x1 = 10.$$

It's Maggie's sequence. Whoa!

puzzle #2

What's my prediction for the next number in Maggie's sequence?

About the Author: Dr. James Tanton

The NMF Weekly is written by mathematician Dr. James Tanton as a resource for friends and fans of the 2021 National Math Festival.

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