the nmf weekly

Ask your math friend, James

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Circles as Polygons

G'Day!

This is your math friend James. Today I am answering a question from Gaurav.

SHOULD WE CONSIDER A CIRCLE A POLYGON WITH AN INFINITE NUMBER OF SIDES?

This is a great question! I have some thoughts.

First, let's be clear what a polygon is. It's a figure drawn on the page composed of straight line segments, connected end-to-end, and enclosing a single area



A circle, on the other hand, is a figure created by swinging a taut string around one end pinned to the page. It has no straight edges and so is not considered a polygon.



But people have noticed that if you draw a large number of dots on the circle and connect the dots with line segments you get a polygon that approximates the circle. The more dots you choose, closely packed together, the better the approximation.



The approximations are very good. Even with the 50 closely-spaced dots in my picture, I can't tell the difference between the 50-sided polygon and the circle.



But the circle really isn't any one of these polygons.

Nonetheless, we humans often like to imagine we can do beyond-human things. What if we could put an infinite number of dots on the circle, infinitely close to each other. Wouldn't we have then an infinitesided polygon that is such a good approximation to the circle that it really <u>is</u> the circle?

This is hard to get your brain around. But you can see why some people might want to say that a circle is like a "polygon with an infinite number of sides."

puzzle #1

If you draw a circle on a piece of paper, it divides the page into 2 regions: an inside and an outside.

If you draw a circle on the surface of an apple, it also divides the apple surface into 2 regions.

But there is a food item for which a circle drawn on its surface fails to divide its surface into 2 regions. What food item is that? Thinking of a circle this way can often lead to interesting mathematical ideas. For example, here's a curious result from geometry class.

Add to each side of a polygon an additional segment that is 1 cm long. Swing each segment around to sweep out a section of a circle. Then all those sections fit together perfectly to make one full circle!



Check out <u>MATHICAL</u> for awardwinning math books for middleschoolers and teens, the YouTube channel <u>NUMBERPHILE</u> for math videos galore, and <u>MORE MATH!</u> for even more resources. Wowza!

Here's a fun <u>Numberphile video</u> about areas of "lunes" which are shapes created by two intersecting circles. Enyoy!

Do you have a math question for me to answer, or try to answer?

Write to me <u>at the website</u>. Each week I'll pick a new question and give my thoughts on it! If you are someone who wants to think of a circle as a polygon with an infinite number of sides, then you might think this result should be true for circles too.

And do you know what? It is!

The wedges would be "infinitely thin" (whatever that means!) and end up sweeping out a ring around the original circle. But the area of this ring is also sure to be the same as the area of a circle of radius 1, just as for polygons.

Wow!

(If you want to do something crazy, try drawing the picture of wedges for a 50-sided polygon to see why this is believable!)

puzzle #2

Any line through the center of a circle divides the circle into two regions of equal area.



Is there another shape and a special point within it such that <u>any</u> line drawn through that special point is sure to divide the shape into two regions of equal area?

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The NMF Weekly is written by mathematician Dr. James Tanton as a resource for friends and fans of the 2021 National Math Festival.

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