Math and Water

G’Day!

This is your math friend James. Today I am answering a question from Dan.

**CAN YOU DO SOMETHING MATHY WITH WATER?**

The answer is surely YES! How about this for a start?

### puzzle #1

You are standing by a lake and you have at your feet two buckets. They are both completely unmarked but you know that the small bucket holds exactly 3 liters of water and the large bucket holds exactly 4 liters of water.

For some reason, which I cannot explain, you need exactly 1 liter of water.

Here’s one way you can get it.

1. Fill up the 4 liter bucket from the lake.
2. Pour water from the 4 liter bucket into the 3 liter bucket until it is full. That leaves 1 liter of liquid in the big bucket.
3. Empty the 3 liter bucket.

You now have exactly 1 liter of water.

There is another way you could accomplish this task. It’s longer. But can you make sense of this sequence of diagrams? (It involves filling up the 3-liter bucket three times and emptying the 4-liter bucket twice.)

Here, finally, is my puzzle for you!

Suppose, instead, I want exactly 2 liters of water. The picture above shows it is possible to get this. (Do you see it?)

Is there a second way to get exactly 2 liters of water?
I remember reading somewhere as a child that the “hardest” crossword clue (with a five-letter word answer) was this: HIJKLMO.

The answer is “water.” Do you see why?

By the way, I just looked on the internet for “the hardest crossword clue” and this one didn’t come up. I wonder what I was reading as a child? Maybe not everything one reads is true!

While I am reading things off of the internet that might or might not be true... I also read that

"70% OF A HUMAN BRAIN IS WATER"

Hmm. What does that mean?

- **Is this a statement about volume?** (If we took out all the water from my brain, would what’s left be just 30% as big?)
- **Is this a statement about weight?** (Maybe my brain is composed of a tiny amount of very heavy matter and is mostly water?)
- **Is this a statement about counting molecules?** Do 70% of the molecules that make the matter of my brain turn out to be water molecules (H2O)?
- **Are all three of these interpretations the same or different?**

I guess I am learning I often don’t know what people mean when they just state “facts” like these!

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**puzzle #2**

I have two buckets the same size. The left one is completely full and the right one is empty.

A "move" consists of pouring a third of the contents of one bucket into the other (leaving two-thirds of the contents of that bucket behind). One can pour either from the left bucket into the right, or from the right bucket into the left.

Is it possible, after some number of moves, to have the left bucket exactly one-quarter full?

Do you have a math question for me to answer, or try to answer? Write to me at the website. Each week I’ll pick a new question and give my thoughts on it!

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Check out MATHICAL for award-winning math books for middle-schoolers and teens, the YouTube channel NUMBERPHILE for math videos galore, and MORE MATH! for even more resources. Wowza!

About 71% of the Earth’s surface is water. Most of that is salt water. Here's a Numberphile video about river water. It asks: Where does it go?

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About the Author: Dr. James Tanton

The NMF Weekly is written by mathematician Dr. James Tanton as a resource for friends and fans of the 2021 National Math Festival.

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