the nmf weekly

Ask your math friend, James

globalmathproject.org/nmf-weekly • ISSUE 7 • November 7, 2020

Math and Water PUZZLE SOLUTIONS

Here are the answers to the puzzles from last time.

puzzle #1

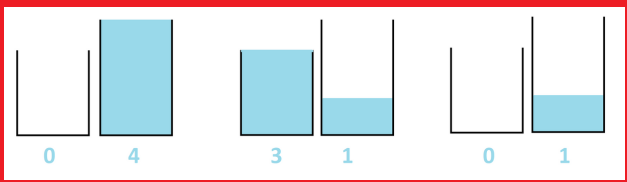
You are standing by a lake and you have at your feet two buckets. They are both completely unmarked but you know that that the small bucket holds exactly 3 liters of water and the large bucket holds exacty 4 liters of water.

For some reason, which I cannot explain, you need exactly 1 liter of water.

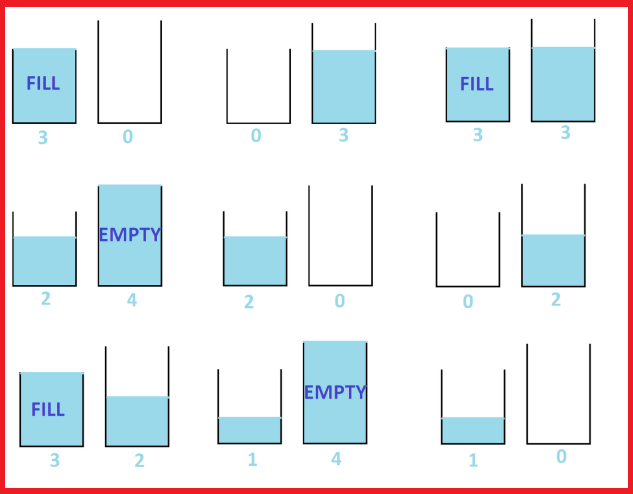
Here's one way you can get it.

1. **Fill up the 4 liter bucket from the lake.**
2. **Pour water from the 4 liter bucket into the 3 liter bucket until it is full. That leaves 1 liter of liquid in the big bucket.**
3. **Empty the 3 liter bucket.**

You now have exactly 1 liter of water.



There is another way you could accomplish this task. It's longer. But can you make sense of this sequence of diagrams? (It involves filling up the 3-liter bucket three times and emptying the 4-liter bucket twice.)



Here, finally, is my puzzle for you!

Suppose, instead, I want exacty 2 liters of water. The picture above shows it is possible to get this. (Do you see it?)

Is there a second way to get exactly 2 liters of water?

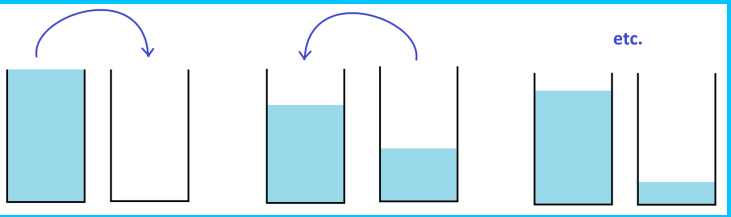
ANSWER TO PUZZLE 1:

Here's a second way. (Hopefully it is okay if I don't draw pictures.)

0,0 → 0,4 → 3,1 → 0,1 → 1,0 → 1,4 → 3, 2 → 0, 2

puzzle #2

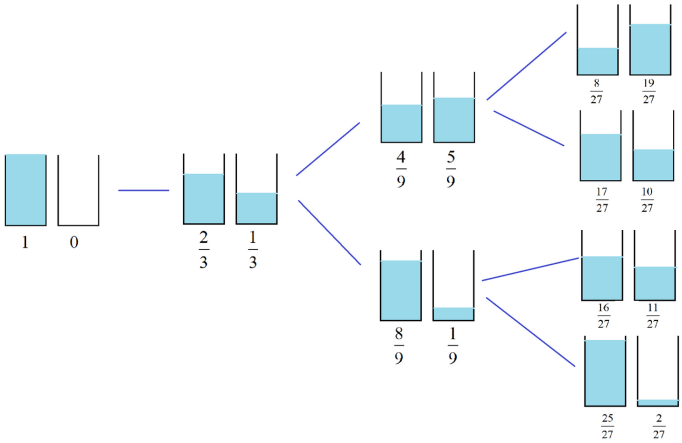
I have two buckets the same size. The left one is completely full and the right one is empty.



A "move" consists of pouring a third of the contents of one bucket into the other (leaving two-thirds of the contents of that bucket behind). One can pour either from the left bucket into the right, or from the right bucket into the left.

Is it possible, after some number of moves, to have the left bucket exactly one-quarter full?

ANSWER TO PUZZLE 2:



Start drawing a diagram of all the moves you can make to see some of the fractions of liquid that arise.

After one move we have fractions of liquid in terms of thirds. After a second move, no matter which choice we make, we have fractions of liquid in terms of ninths. After a third move, no matter which choices we make, we have fractions of liquid in terms of twenty-sevenths.

Since we're always pouring a third of the liquid we have, after a fourth move we'll have fractions of liquid in terms of 81ths (since 3 x 27 = 81). After the next move we'll have fractions in terms of 243rds (since 3 x 81 = 243), and so on.

The fraction 1/4 cannot be written as an equivalent fraction in terms of ninths. (Why not?) It cannot be written as an equivalent fraction in terms of 27ths. (Why not?). Nor 81ths. Nor 243ths. Nor 729th. Nor any fraction with a denominator a product of just threes. We will never see a container exactly one quarter full.

About the Author: Dr. James Tanton

The NMF Weekly is written by mathematician Dr. James Tanton as a resource for friends and fans of the 2021 National Math Festival.

Learn more at globalmathproject.org/nmf-weekly & nationalmathfestival.org