globalmathproject.org/nmf-weekly • ISSUE 5 • October 12, 2020

Math and Mirrors PUZZLE SOLUTIONS

Here are the answers to the puzzles from last time.

puzzle #1

Stand in front of a big mirror mounted on a wall.

Using some painters' tape — something that doesn't leave a residue when removed — or a washable marker designed for writing on glass, mark two horizontal lines on the mirror: one at the bottom of your chin and one at the top of your forehead. (I tried to take a photo of me doing this.)

Now, back away from the mirror. Next, walk closer to the mirror.

What weird thing do you notice?

Can you explain what you observe?

ANSWER TO PUZZLE 1:

Did you notice that no matter how far you stood from the mirror the tape at the top of your forehead always appears to be at the top of your forehead and the tape at the bottom of your chin always appears at the bottom of your chin? Isn't that weird?

This would still be true even if you stood a mile away from the mirror. (Though you would have a hard time seeing the mirror and your reflection from a mile away!)

To see why, look at a ray of light reflected off your forehead that bounces off the mirror to then head into your eyes. In the diagram I've drawn this ray of light blue. I've also drawn in two horizontal lines in orange that match eye level and the level of the top of your head.
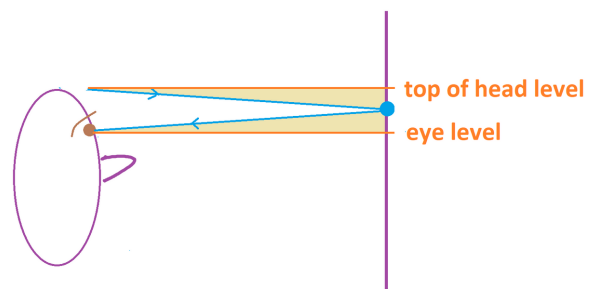
Because rays of light bounce off mirrors to keep angles the same, we see that the two triangles I've shaded are identical in all ways: angles, lengths, shape, and size. (One is just an upside-down version of the other.) This means that that the ray of light is bouncing off the point on the mirror halfway between the top-of-your-head level and eye level. This is where you put a piece of tape.

In the same way, rays of light coming from your chin bounce off the mirror halfway between the level of your chin and eye level. This is where you put the second piece of tape.

And you can see that picture makes no reference to how far away from the mirror you are! You will always have tape at these same half-way points to match where you see the top of your head and the bottom of your chin.

puzzle #2

A lazy dog is at point A on one side of a wall and wants to walk to point B on the same side of the wall. But being a dog, it wants to stop along the way at the wall to sniff at the wall.

The dog could walk straight to the wall and then head to B. Or the dog could walk to the point directly below B and then head to B. Or the dog could walk to some intermediary point between these two extremes, sniff the wall there, and then walk to B.
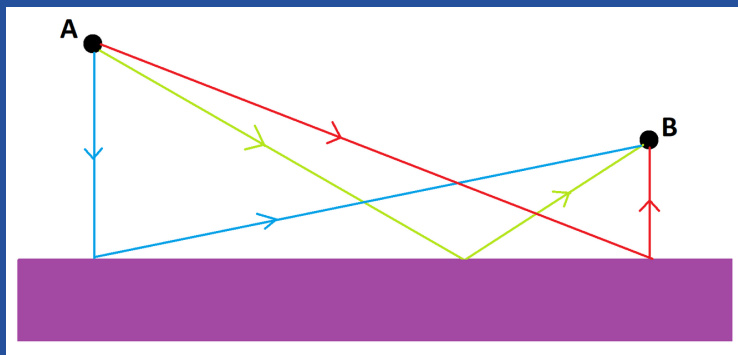

Being lazy, the dog wants to walk the shortest path possible.

Where exactly should the dog sniff the wall so that its path from A to that point along the wall and then to B is as short as possible?
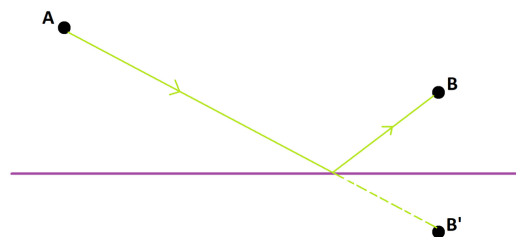
ANSWER TO PUZZLE 2:

Imagine a point B' the other side of the wall the same distance from the wall as the point B is.

Then any path from A to the wall to B is matched with a path of equal length from A to the wall to B'.
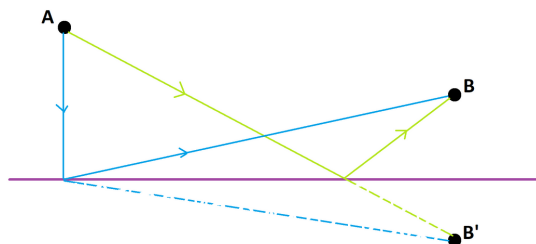
The shortest path from A to B' is the straight-line path.

This must then give the shortest path from A to the wall to the point B.

So the dog should walk the same path as a ray of light does "reflecting" off the wall as though the wall was a mirror!

(In fact, to make this easy for the dog we could polish the wall until it really was a mirror. Then all the dog would have to do is walk from A straight towards the reflection of the point B in the mirror until it hits the wall, and then head to the real point B.)

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The NMF Weekly is written by mathematician Dr. James Tanton as a resource for friends and fans of the 2021 National Math Festival.

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