G’Day!

This is your math friend James.

Today I am answering a "why" math question from Johan K. in South Africa. He asked:

"WHY IS NEGATIVE TIMES NEGATIVE POSITIVE?"

It’s an age-old question that most everybody wants answered.

I remember being told in school to think of -1, (negative one) as "the opposite of one." So I imagined 1 as a pile of sand and -1 as the opposite of a pile, which would be a hole. Put a pile and a hole together, you get nothing, zero.

Then my teacher said: "So -(−1), that is, negative negative one, would be the opposite of the opposite of one. This is back to 1." I imagined ...

And then the conversation stopped. I realized then that this wasn’t a conversation about why negative times negative is positive. There was no mention of multiplication.

But at least it was fun to think in terms of piles and holes for some basic arithmetic.

Young James liked to think in terms of piles and holes.

For example, for him, 4 + (-2) was "four piles and two holes," which makes two piles, +2.

And - - 3 was "the opposite of the opposite of three piles," which is back to being three piles.

How do you think Young James thought about each of these expressions?

a) −−− −3
b) −−−−−−−−−−−−−−−−−−−−−−3
c) 6 + (-2) +1
d) 5 + (-7)
e) 2 + (-5) + (-10) + 11 + 6 + (-4)
Are there times of day when the two hands—
the minute hand and the hour hand—lie in
opposite directions?

In school I was taught that in the world of counting
numbers multiplication appears as repeated addition.
Thus 4x5, for instance, is “four groups of five” (back in
Australia we said “four lots of five.”) So:

\[ 4 \times 5 = 5 + 5 + 5 + 5 = 20. \]

I always thought it astounding that we could switch the
order of the multiplication and be sure to get the same
answer.

\[ 5 \times 4 = \text{"five groups of four"} \]
\[ = 4 + 4 + 4 + 4 + 4 = 20. \]

That sounds like another "why" question.

Why should 35 x 878 equal 878 x 35? Is it obvious
that 35 groups of 878 apples gives the same count
of apples as 878 groups of just 35 apples?

(And even though (-4) x 5 makes no sense (“negative four
groups of five”) we would say “just switch the order!”
(Ooh! I guess we *are* believing negative numbers
behave just the same was as positive ones.)

\[ (-4) \times 5 = 5 \times (-4) = \text{"five groups of negative four"} \]
\[ = -4 + -4 + -4 + -4 + -4 = -20. \]

So we can handle positive times positive, and postive
times negative, and negative times positive. But what
about (-4) x (-5), say? It’s negative times negative.
Switching the order doesn’t help this time! However ...
look at puzzle 2.

\[ 10 \times 5 = \]
\[ \begin{array}{cc}
100 & 50 \\
60 & 30 \\
\end{array} \]
\[ 20 \times -5 = \]
\[ \begin{array}{cc}
200 & -50 \\
120 & -30 \\
\end{array} \]

\[ 15 \times 16 = 100 + 50 + 60 + 30 = 240 \]
\[ 15 \times 16 = 200 + (-50) + 120 + (-30) = 240 \]

\[ 10 \times 5 = \]
\[ \begin{array}{cc}
200 & 100 \\
-40 & -20 \\
\end{array} \]
\[ 20 \times -5 = \]
\[ \begin{array}{cc}
400 & -100 \\
-80 & ? \\
\end{array} \]

\[ 15 \times 16 = 200 + 100 + (-40) + (-20) = 240 \]
\[ 15 \times 16 = 400 + (-100) + (-80) + ? = 240 \]

But if we believe that multiplication with negative
numbers works just the same way as with positive
counting numbers, then we would say:

\[ 4 \times (-5) = \text{"four groups of negative five"} \]
\[ = -5 + -5 + -5 + -5 = -20. \]

OTHER RESOURCES

Check out MATHICAL for award-winning
math books for middle-schoolers and
teens, the YouTube channel NUMBERPHILE
for math videos galore, and MORE MATH!
for even more resources. Wowza!

If you want to play with negative numbers and
subtraction in fun ways, check out EXPLODING
DOTS listed in MORE MATH! here.

About the Author: Dr. James Tanton

The NMF Weekly is written by mathematician
Dr. James Tanton as a resource for friends and
fans of the 2021 National Math Festival.
Learn more at globalmathproject.org/nmf-
weekly & nationalmathfestival.org

WRITE TO ME

Do you have a math question you’d like
me to answer, or try to answer?

Have an adult help you to write to me at the
website. Each week I’ll pick a new question
and give my thoughts on it! Plus I’ll give my
solutions to the most recent puzzles.