the nmf weekly

Ask your math friend, James

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Negative Numbers

G'Day!

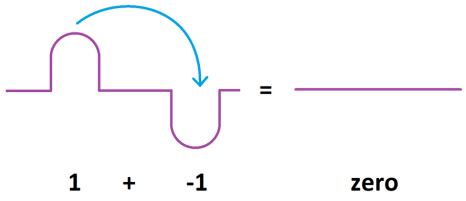
This is your math friend James.

Today I am answering a "why" math question from Johan K. in South Africa. He asked:

"WHY IS NEGATIVE TIMES NEGATIVE POSITIVE?"

It's an age-old question that most everybody wants answered.

I remember being told in school to think of -1, (negative one) as "the opposite of one." So I imagined 1 as a pile of sand and -1 as the opposite of a pile, which would be a hole. Put a pile and a hole together, you get nothing, zero.

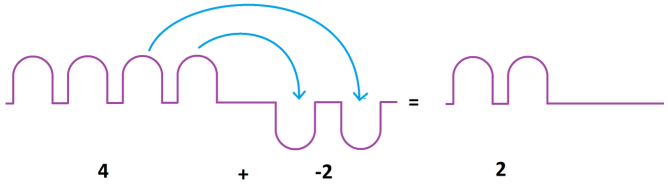


Then my teacher said: "So -(-1), that is, negative negative one, would be the opposite of the opposite of one. This is back to 1." I imagined ...

1 = a pile
-1 = the opposite of a pile = a hole
-(-1) = the opposite of a hole = a pile = 1

And then the conversation stopped. I realized then that this wasn't a conversation about why negative times negative is positive. There was no mention of multiplication.

But at least it was fun to think in terms of piles and holes for some basic arithmetic.



puzzle #1

Young James liked to think in terms of piles and holes.

For example, for him, 4 + (-2) was "four piles and two holes," which makes two piles, +2.

And - - 3 was "the opposite of the opposite of three piles," which is back to being three piles.

How do you think Young James thought about each of these expressions?

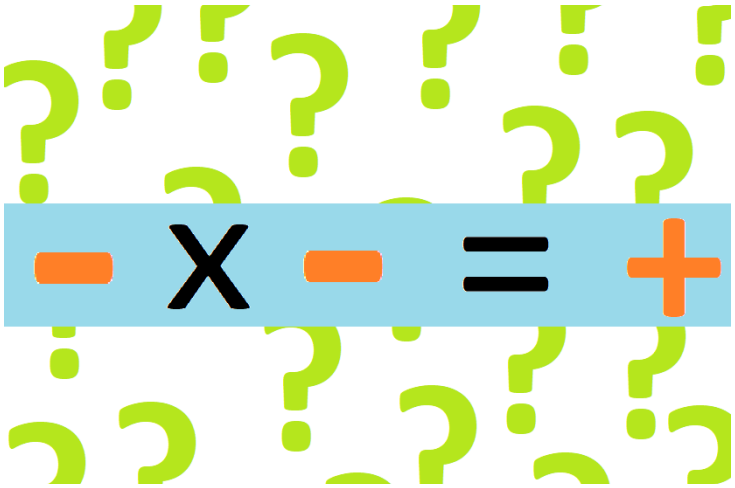
a) - - - 3

b) - 3

c) 6 + (-2) + 1

d) 5 + (-7)

e) 2 + (-5) + (-10) + 11 + 6 + (-4)



In school I was taught that in the world of counting numbers multiplication appears as repeated addition. Thus 4x5, for instance, is "four groups of five" (back in Australia we said "four lots of five.") So:

$$4 \times 5 = 5 + 5 + 5 + 5 = 20.$$

I always thought it astounding that we could switch the order of the multiplication and be sure to get the same answer.

$$5 \times 4 = \text{"five groups of four"} = 4 + 4 + 4 + 4 + 4 = 20.$$

That sounds like another "why" question.

Why should 35 x 878 equal 878 x 35? Is it obvious that 35 groups of 878 apples gives the same count of apples as 878 groups of just 35 apples?

(It doesn't at all seem "obvious" to me!)

But if we believe that multiplication with negative numbers works just the same way as with positive counting numbers, then we would say:

$$4 \times (-5) = \text{"four groups of negative five"} = -5 + -5 + -5 + -5 = -20.$$

And even though (-4) x 5 makes no sense ("negative four groups of five") we would say "just switch the order!" (Ooh! I guess we *are* believing negative numbers behave just the same was as positive ones.)

$$(-4) \times 5 = 5 \times (-4) = \text{"five groups of negative four"} = -4 + -4 + -4 + -4 + -4 = -20.$$

So we can handle positive times positive, and postive times negative, and negative times positive. But what about (-4) x (-5), say? It's negative times negative. Switching the order doesn't help this time! However ... look at puzzle 2.

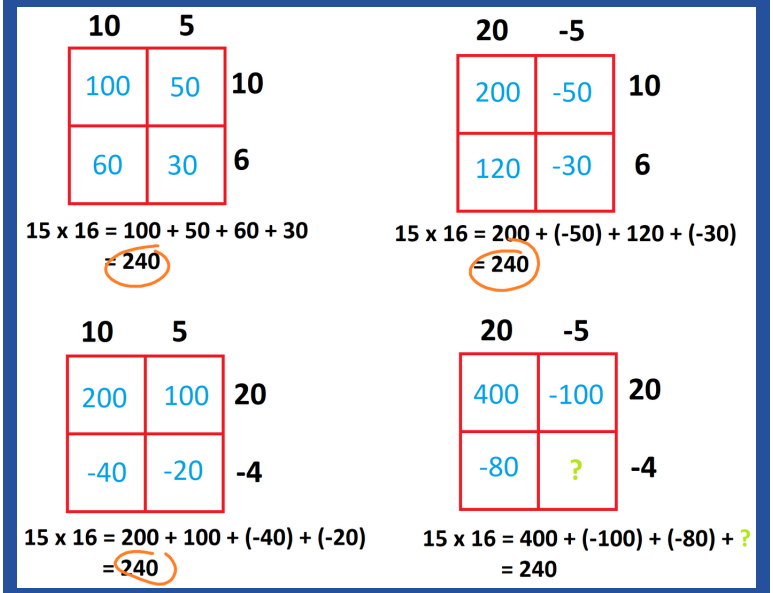
WRITE TO ME

Do you have a math question you'd like me to answer, or try to answer?

Have an adult help you to write to me at the website. Each week I'll pick a new question and give my thoughts on it! Plus I'll give my solutions to the most recent puzzles.

puzzle #2

People do believe that negative numbers follow all the same rules of arithmetic as positive numbers. In which case, what does the fourth picture below say is the logical value of (-4) x (-5)? (Here we are computing 15 x 16 four different ways.)



OTHER RESOURCES

Check out MATHICAL for award-winning math books for middle-schoolers and teens, the YouTube channel NUMBERPHILE for math videos galore, and MORE MATH! for even more resources. Wowza!

If you want to play with negative numbers and subtraction in fun ways, check out EXPLODING DOTS listed in MORE MATH! here.

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The NMF Weekly is written by mathematician Dr. James Tanton as a resource for friends and fans of the 2021 National Math Festival. Learn more at globalmathproject.org/nmf-weekly & nationalmathfestival.org