the nmf weekly

Ask your math friend, James

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Negative Numbers: PUZZLE SOLUTIONS

Here are the answers to the puzzles from last time.

puzzle #1

Young James liked to think in terms of piles and holes.

For example, for him, 4 + (-2) was "four piles and two holes," which makes two piles, +2.

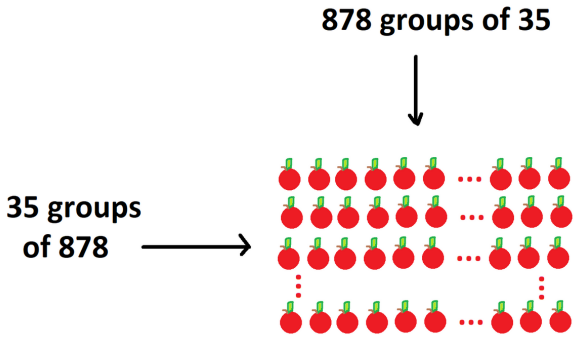
And - - 3 was "the opposite of the opposite of three piles," which is back to being three piles.

How do you think Young James thought about each of these expressions?

a) - - - 3
b) - 3
c) 6 + (-2) + 1
d) 5 + (-7)
e) 2 + (-5) + (-10) + 11 + 6 + (-4)

By the way ... 35 groups of 878 apples can be seen to be the same as 878 groups of 35 apples.

Imagine arranging your 35 groups of 878 apples in rows to make a rectangular array of apples.



By looking at the ray column-wise, we then see that we also have a picture of 878 groups of 35 apples. It is the same count of apples no matter how you look at the array, so, philosophically, 35 x 878 must give the same answer as 878 x 35.

(Wow! Without even knowing the answer to 35x878
I know that these two values just must be the same.)

ANSWER TO PUZZLE 1:

a) The opposite of the opposite of the opposite of three piles is ... three holes. This is -3.

b) We have 21 opposites of three piles here. That is again three holes: -3.

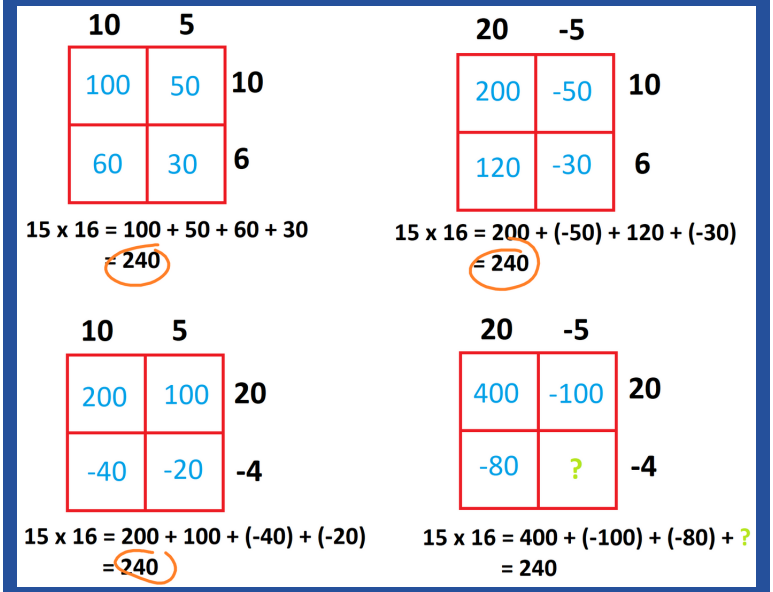
c) 6 piles and 2 holes and 1 pile is (after filling the two holes) 5 piles: +5.

d) 5 piles and 7 holes is (after filling up the holes) 2 holes: -2.

e) Can you see all the piles here fill all the holes? The answer is: 0.

puzzle #2

People do believe that negative numbers follow all the same rules of arithmetic as positive numbers. In which case, what does the fourth picture below say is the logical value of (-4) x (-5)? (Here we are computing 15 x 16 four different ways.)



ANSWER TO PUZZLE 2:

The top left picture computes 15x16 as

(10 + 5) times (10 + 6)

and we see the answer 240 results.

The top right picture computes 15x16 as

(20 + -5) times (10 + 6)

and we see the answer 240 results.

The bottom left picture computes 15x16 as

(10 + 5) times (20 + -4)

and we see the answer 240 results.

The bottom right picture computes 15x16 as

(20 + -5) times (20 + -4)

and the answer 240 just better be the result! But for this to be the case, (-4)x(-5) just has to equal +20.

- It is very very hard to find a practical "real world" model that illustrates why mathematics insists that negative times negative be positive. The truth is that it is simply a logical consequence of us insisting that negative numbers — and all numbers, in fact — should follow the same basic arithmetic rules as the positive numbers.

- Surprisingly, this insistence has proved to be very helpful: the mathematics that results does accurately model so much of the real world! But there are logical quirks that just have to be accepted as small prices to pay. One is the non-intuitive idea that negative times negative has to be positive.

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The NMF Weekly is written by mathematician Dr. James Tanton as a resource for friends and fans of the 2021 National Math Festival.

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