EXPLODING DOTS CERTIFICATION PROBLEM SET

The Global Math Project and its partners Edfinity and Scolab are pleased to provide educators the opportunity to earn official recognition for conducting professional development on the topic of EXPLODING DOTS!

How It Works

1. Review the following eight EXPLODING DOTS experiences.

   Experience 1 - The Machines
   Experience 2 – Insight
   Experience 3 - Addition and Multiplication
   Experience 4 - Subtraction
   Experience 5 - Division
   Experience 6 - All Bases, All at Once, Polynomials
   Experience 7 - Infinite Sums
   Experience 8 – Decimals

2. When you are ready go to this Edfinity site to complete the Certification Problem Set of 60 multiple-choice questions.

   You can preview and practice all the questions in this document!

3. Attempt this Certification Problem Set as many times as you like online, save your work and come back to do in stages or do it one sitting.

4. Earn a score of 85% or more, and you will earn a letter of certification.

5. A personalized letter of certification will be issued by the Global Mathematics Project to acknowledge the completion of **8 hours** of online training. Certificates are typically issued within two weeks of each calendar quarter.

6. This self-certification program costs US$6.99 per participant to help the Global Mathematics Project sustain its work. Payments are non-refundable. If this is hardship for you, contact us at info@globalmathproject.org and we will help.

7. Although the Global Mathematics Project cannot provide any assurance that your certificate will be honoured by your school or local education district as proof of professional development work (administrative rules on this vary significantly across the globe), you will have done good thinking and valuable work.

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THE QUESTIONS

EXPERIENCE 1: THE MACHINES

Question 1: How many 1s appear in the $1 \leftarrow 2$ machine code for the number twenty?

(A) One
(B) Two
(C) Three
(D) Four

Question 2: Which number has code 1011 in a $1 \leftarrow 2$ machine?

(A) Eleven
(B) Thirteen
(C) Fifteen
(D) None of these

Question 3: The code for a number in a $1 \leftarrow 3$ machine is $4\,|\,0\,|\,3\,|\,2$, but this code is not "stable." Which of the following is the stable version of this code?

(A) $1\,|\,0\,|\,0\,|\,3\,|\,2$
(B) $4\,|\,1\,|\,0\,|\,2$
(C) $1\,|\,0\,|\,0\,|\,0\,|\,2$
(D) $1\,|\,0\,|\,1\,|\,0\,|\,2$
(E) None of these

Question 4: Which of the following is the code for the number fifty in a $1 \leftarrow 7$ machine?

(A) $1\,|\,1$
(B) $1\,|\,0\,|\,1$
(C) $1\,|\,0\,|\,0\,|\,1$
(D) $1\,|\,0\,|\,0\,|\,0\,|\,1$
(E) None of these. They are all $1 \leftarrow 2$ machine codes!

Question 5: When Edna put five-hundred dots in $1 \leftarrow 9$ machine, she got the code $6\,|\,0\,|\,1\,|\,5$. How many dots would Shivram have to put into a $1 \leftarrow 10$ machine to see this same code appear?

(A) 500
(B) 615
(C) Neither of these

Question 7: For the list of statements below, check those that are always true.

(A) The $1 \leftarrow 2$ machine code for an even numbers ends with a 0.
(B) The $1 \leftarrow 3$ machine code for a number that is a multiple of three ends with a 0.
(C) The $1 \leftarrow 5$ machine code for a number that is a multiple of twenty-five ends with a zero.
(D) The $1 \leftarrow 5$ machine code for a number that is a multiple of twenty-five ends with two zeros.
(E) The $1 \leftarrow 5$ machine code for a number that is a multiple of twenty-five ends with three zeros.

Question 8: James, the creator of *Exploding Dots*, always starts his presentations on the topic with the story of the $1 \leftarrow 2$ machine and the $1 \leftarrow 3$ machine to lead up to the $1 \leftarrow 10$ machine, even with high school students and university students, because of which of the following reasons? Check all that apply. (HINT: Only one does not apply.)

(A) Starting at the very beginning provides emotional safety for all and welcomes everyone to story.

(B) Mathematics is a story and students rarely get to see and experience the full story.

(C) It is always good to revisit beginning ideas and see them in a new light: one notices subtleties and nuances and one's understanding of a mathematics deepens as a whole.

(D) It is important not to rush so as to focus on deep understanding and to teach thinking.

(E) It's a story that induces joy and delight and brings context. (One should engage in joy and delight and provide context.)

(F) James likes pineapple on pizza.

EXPERIENCE 2: INSIGHT

Question 8: Which number has 10000000000 as its $1 \leftarrow 2$ machine code?

(A) 64
(B) 128
(C) 256
(D) 512
(E) 1024
(F) 2048

Question 9: How many 1s are in the binary code for the number sixty-three?

(A) Four
(B) Five
(C) Six
(D) Seven
(E) Eight

Question 10: A number has $1 \leftarrow 5$ machine code $1 \mid 3 \mid 4$. Find that number and then check each of the statements below that is true about that number.

(A) The number is even.
(B) The number is divisible by three.
(C) The number is divisible four.
(D) The number is divisible by five.
(E) The number is divisible by six.

Question 11: Check each statement below that is true.

(A) In English, individual fingers and toes are called “digits.”
(B) Humans are usually born with ten fingers and humans were, by-and-large, drawn to creating a base-ten number system.
(C) Some human cultures have used base-twenty number systems.
(D) Computers, based on electrical switches that are either on or off, work on a base-two number system.

Question 12: If one were to put 632 dots into the rightmost box of a $1 \leftarrow 10$ machine and perform all the individual explosions until the code $6 \mid 3 \mid 2$ appears, how many explosions in total would occur?

(A) 11
(B) 65
(C) 69
(D) 632
(E) None of these

Question 13: Characters in J. R. R. Tolkien's famous trilogy "The Lord of the Rings" spoke with a base-ten number system but would, on occasion, mention a number such as "eleventy-six." This makes good sense in a $1 \leftarrow 10$ machine. What number is "eleventy-six"?

(A) 16
(B) 17
(C) 66
(D) 116

Question 14: Some teachers say that when ten dots fuse together in a $1 \leftarrow 10$ machine to become one dot one place to the left, that new dot should be drawn ten times as large. What is the most likely reason that the *Exploding Dots* program chose <u>not</u> to draw larger dots?

(A) When we write numbers such as 222 for two-hundred-twenty-two we do not draw digits ten-times as large. The *Exploding Dots* program is setting the stage for teachers to have this conversation with their students.

(B) There was no pedagogical reason.

Question 15: Which number has code $2 \mid 1 \mid 0 \mid 1 \mid 2$ in a $2 \leftarrow 3$ machine?

(A) 16
(B) 17
(C) 18
(D) 19
(E) 20

Question 16: Consider codes from a $1 \leftarrow 2$ machine.

The code 11 with **two** ones (and no zeros) represents the number **three**: a prime number.
The code 111 with **three** ones represents the number **seven**: a prime number.
The code 11111 with **five** ones represents the number **thirty-one**: a prime number.

Does the code 1111111 with **seven** ones represent a prime number?

> **Comment:** This pattern, alas, does *not* continue: the number represented by **eleven** ones is a composite number. (What number is it?) Any prime number with binary code nothing but ones is today called a *Mersenne Prime* and it is known that any such prime must have a prime number of ones in its binary code. (But as 11111111111 shows, the reverse need not be true.) Here's a famous unsolved question in mathematics: *Are there infinitely many examples of Mersenne primes?* No one knows!

(A) No. I worked out what number has code 1111111 and can see it is *not* a prime number.

(B) Yes. I worked out what number has code 1111111, I typed on the internet "Is this number prime?" and it said it was!

EXPERIENCE 3: ADDITION AND MULTIPLICATION

Question 17: Here's a long addition problem completed from left to right.

$$\begin{array}{rcccccccccc} & 4 & 5 & 7 & 8 & 4 & 0 & 6 & 1 & 9 \\ + & 2 & 8 & 4 & 8 & 6 & 9 & 1 & 9 & 3 \\ \hline = & 6 & 13 & 11 & 16 & 10 & 9 & 7 & 10 & 12 \end{array}$$

Adjust the final line by conducting all the explosions necessary to obtain an answer society understands. How many times does the digit 7 appear in this final version of the answer?

(A) No times
(B) Once
(C) Twice
(D) Thrice (that is, three times)

Question 18: When asked to find a number which, when added to 3794 gives the answer 8888, Ariana wrote $5\,|\,1\,|\,-1\,|\,4$.

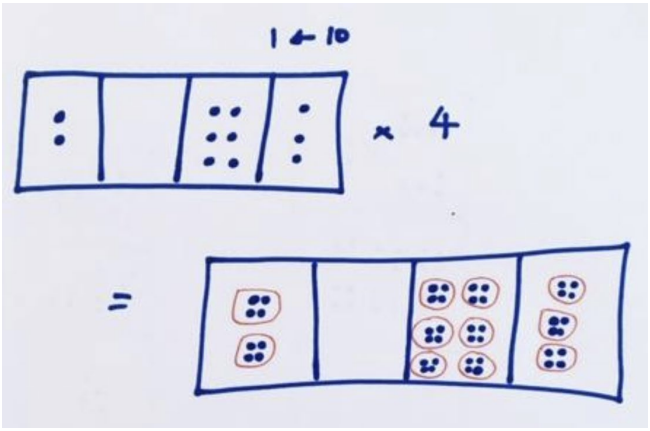
$$\begin{array}{rr} & 3\,7\,9\,4 \\ + & *\,*\,*\,* \\ \hline = & 8\,8\,8\,8 \end{array}$$

Which of the following is the best reaction to Ariana's work?

(A) "Oh. That is brilliant and wonderful, and clearly correct. Can we work out now what number $5\,|\,1\,|\,-1\,|\,4$ must be?"

(B) "Oh. That is brilliant and wonderful, and clearly correct. But there is no way we can possibly make sense of that answer as a number."

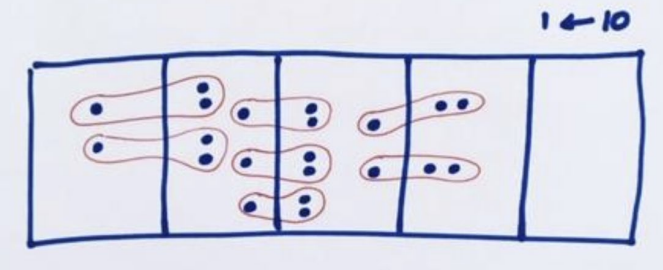
Question 19: The following is a picture of which mathematical computation?



(A) $2063 \times 4 = 8\,|\,0\,|\,24\,|\,12$

(B) Some entirely different computation

Question 20: The following is a picture of which multiplication computation?



(A) 2320×12
(B) 23200×12
(C) 2320×21
(D) 23200×12
(E) Something else entirely

Question 21: Multiplying 1234 by ten gives the answer 12340. Which of the following is the best mathematical explanation for this?

(A) The rule is to add a zero to the end of a number.

(B) The answer is actually $10 \mid 20 \mid 30 \mid 40$ and then explosions give 12340. (This gives the appearance we've just added a zero to the end of the number.)

(C) Mathematics is just mysterious.

Question 22: When asked to compute 4158×24, Mei wrote something complicated.

$$
\begin{array}{rcccc}
 & 4 & 5 & 1 & 8 \\
\text{x} & & & 2 & 4 \\
\hline
 & 16 & 20 & 4 & 32 \\
 & 80 & 100 & 20 & 160 \\
\hline
= & 96 & 120 & 24 & 192
\end{array}
= 96|120|43|2
$$

She claims this leads to the correct answer of 108432.

Which one of the following statements is true?

(A) Mei's method is incomprehensible and her claim that it leads to the correct answer is false.

(B) Mei's method is incomprehensible and it leads to the correct answer 108432 only by luck in this one case.

(C) Mei's method is mathematically solid, but there is an arithmetic error in her work.

(D) Mei's method is mathematically solid and completely free of errors.

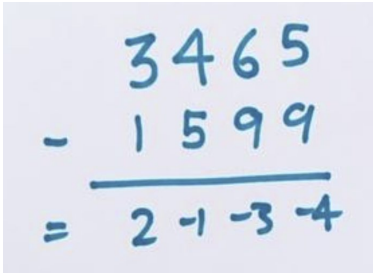
EXPERIENCE 4: SUBTRACTION

Question 23: One can legitimately say that there is no such thing as subtraction: subtraction is just the *addition* of the opposite.

Which of the following is a valid interpretation of $7-10+5-8-2$?

(A) $7+(-10)+5+(-8)+(-2)$
(B) Not the above because mathematics does not allow us to write $+$ and $-$ next to each other.

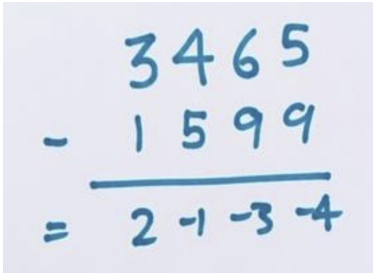
Question 24: Consider the following subtraction problem.



The final answer is equivalent to which of the following? Check all that apply.

(A) $2000-100-30-4$
(B) $1\,|\,9\,|\,-3\,|\,-4$
(C) $1\,|\,8\,|\,7\,|\,-4$
(D) 1866

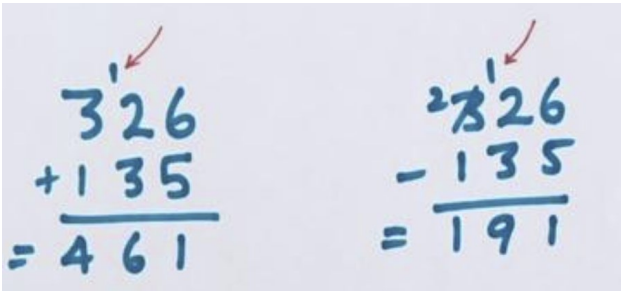
Question 25: Consider the following subtraction problem.



Gaya says that the answer is equivalent to $2\,|\,-1\,|\,-4\,|\,6$ and to $2\,|\,-2\,|\,6\,|\,6$ and to $1\,|\,8\,|\,6\,|\,6$. Is she correct with all three of her claims?

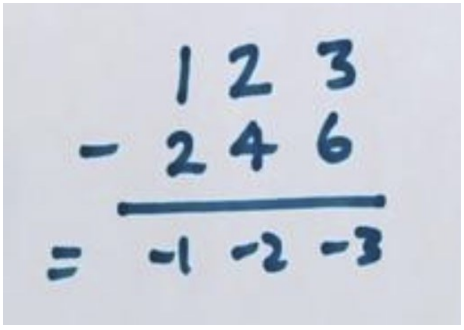
(A) Yes!
(B) No!

Question 26: The traditional algorithms are confusing! Do the little ones in the tens column of each of these problems mean the same thing?



(A) Yes. They both mean "one group of ten."
(B) No. One means "one group of ten" and the other means "ten groups of ten."

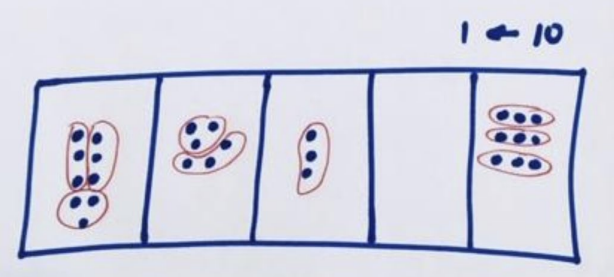
Question 27: The final answer $-1\,|\,-2\,|\,-3$ in this subtraction problem is equivalent to which of the following?



(A) $(-100)+(-20)+(-3)=-123$
(B) Something else.

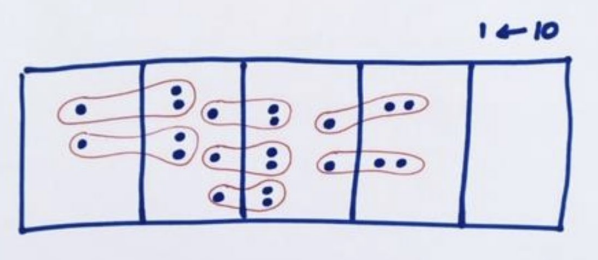
EXPERIENCE 5: DIVISION

Question 28: What does the following picture show?



(A) $96309 = 32103 \times 3$ or equivalently that $96309 \div 3 = 32103$
(B) Something else entirely.

Question 29: What multiplication computation does the following picture show?



(A) $27840 = 2320 \times 12$
(B) $27840 = 23200 \times 12$
(C) Something else entirely.

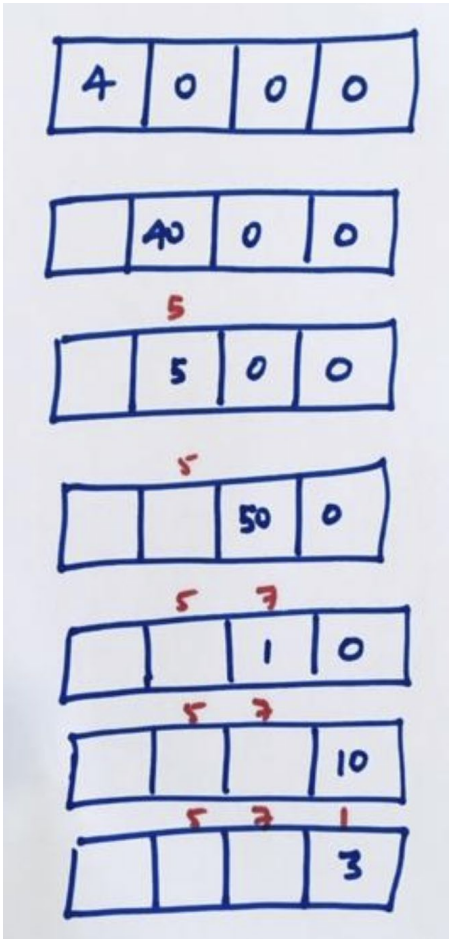
Question 30: It turns out that $32033 \div 103$ equals 311. When you verify this via *Exploding Dots* one must perform some unexplosions. What is the least number of unexplosions one needs?

(A) One
(B) Two
(C) Three

Question 31: Draw dots and boxes to compute $895 \div 42$. What remainder do you see?

(A) A remainder of 3.
(B) A remainder of 13.
(C) A remainder of 23.

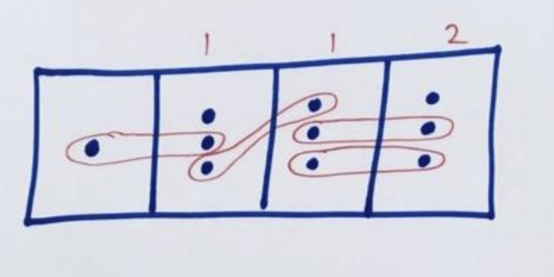
Question 32: When asked to compute $4000 \div 7$ Rakhi produced the following sequence of diagrams and said that the answer is 571 with a remainder of 3, which is correct.



Look at the diagrams. When you feel you can honestly make sense of what Rakhi was doing check the box below.

(A) I can honestly make sense of what Rakhi was doing.

Question 33: When Kiran was asked to compute $1333 \div 11$ he drew the following picture and said that the answer is 112 with a remainder of 101.



His teacher pointed out that his answer is mathematically correct, but that most people would expect a remainder smaller than 11 when dividing by 11.

What could Kiran do to see the answer 121 with a remainder of 2?

(A) He could perform two unexplosions to see the answer $1\,|\,1\,|\,11$ with a remainder of 2.

(B) There is nothing Kiran can do but to restart the problem and try to see the answer $121\ R\ 2$ right away.

Question 34: In dividing 3450 by ten, one gets the answer 345 because of which mathematical reason?

(A) The rule is that one just takes off the final zero when dividing by ten.

(B) 3450 can be rewritten, with unexplosions, as $30\,|\,40\,|\,50$, which can now be seen as 345×10.

(C) Mathematics is just mysterious and can't be explained.

Question 35: Which of the following is true of the *Exploding Dots* program?

(A) The *Exploding Dots* program teaches the box-and-dots method because it believes students should draw tedious diagrams each and every time they are asked to compute a division problem.

(B) The *Exploding Dots* program teachers the box-and-dots method so that students can see and understand the division process but encourages them to follow less tedious pencil-and-paper methods in everyday practice.

EXPERIENCE 6: ALL BASES, ALL AT ONCE

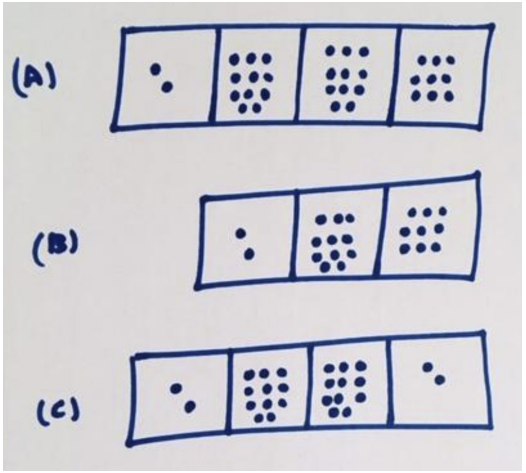
Question 36: Compute, via dots and boxes, the division problem $\left(2x^4+x^3+4x^2+4x+1\right)\div\left(2x+1\right)$ to get an answer of the form ax^3+bx^2+cx+d with one of the values a, b, c, or d equal to zero. Which one is zero?

(A) a
(B) b
(C) c
(D) d

Question 37: For what value of x is the answer to $\dfrac{3x^4+2x^3+7x^2+4x+2}{x^2+2}$ equal to 86?

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

Question 38: Which of the following is a picture of $\left(x^2+x+1\right)\times\left(2x+9\right)$?



(A) Picture A
(B) Picture B
(C) Picture C
(D) None of these

Question 39: Draw the dots and boxes picture that shows that $\frac{x^5 - 3x^3 - x^2 + 2x + 1}{x^2 - x - 1}$ equals $x^3 + x^2 - x - 1$. When you honestly see this answer for yourself, check the box below.

(A) I honestly see this answer for myself.

Question 40: $\frac{x^8 - 1}{x^2 + 1}$ equals which of the following?

(A) $x^6 + x^4 + x^2 + 1$
(B) $x^6 - x^4 + x^2 - 1$
(C) $x^6 + x^4 - x^2 - 1$
(D) $-x^6 + x^4 - x^2 + 1$
(E) None of these.

Question 41: Compute $\frac{x^3 - 4x^2 + 6x - 4}{x - 2}$ to get an answer in the form $ax^2 + bx + c$. What is the value of $a + b + c$?

(A) 0
(B) 1
(C) -1
(D) 2
(E) -2

Question 42: What is the remainder when x^6 is divided by $x^2 - 1$?

(A) 1
(B) -1
(C) x
(D) $-x$
(E) None of these

Question 43: ax^3+bx^2+cx+d divided by $x-1$ equals $ax^2+(a+b)x+(a+b+c)+\dfrac{a+b+c+d}{x-1}$.
(Draw a dots and boxes picture to convince yourself that this is true.) By choosing $x=10$, this illustrates which famous divisibility rule in mathematics?

(A) A number is divisible by two if its final digit is 0, 2, 4, 6, or 8.
(B) A number is divisible by four if its final two digits represent a number divisible by four.
(C) A number is divisible by five if its final digit is 0 or 5.
(D) A number is divisible by nine if the sum of its digits is divisible by nine.
(E) A number is divisible by ten it its final digit is zero.

Question 44: $1787276^{1003}-1$ is a multiple of 1787275. True or false?

(A) True
(B) False

Question 45: It is worthwhile to teach polynomial division to students because ….

(A) It is an important skill that adults use most every day in their lives.

(B) It, in and of itself, is not a meaningful enterprise that most students will ever need to know, but it is an opportunity to teach self-reliant thinking and the confidence to take independent action to solve problems in mathematics (and in life!).

EXPERIENCE 7: INFINITE SUMS

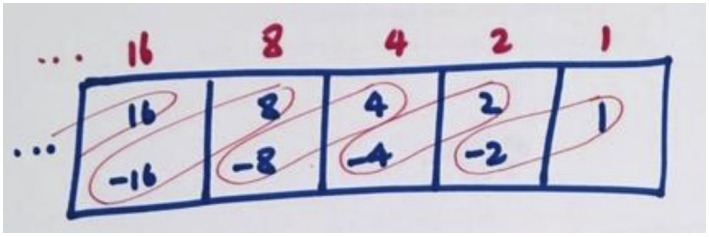
Question 46: Compute $\dfrac{1}{1-x^2}$. Do you get the infinite sum of the even powers of x?

(A) Yes, I do.
(B) No. Not quite.

Question 47: Compute $\dfrac{x}{1+x^2}$. Do you get the infinite sum of the even powers of x?

(A) Yes.
(B) No.

Question 48: When Natasa computed $\frac{1}{1-2x}$ she drew this picture.



She claimed that this means that $\frac{1}{1-2x} = 1 + 2x + 4x^2 + 8x^3 + 16x^4 + \cdots$.

Is she correct in her thinking that $\frac{1}{1-2x}$ is algebraically equivalent to $1 + 2x + 4x^2 + 8x^3 + 16x^4 + \cdots$?

(A) She is absolutely correct.
(B) She is not correct.

Question 49: It is true that $\frac{1}{1-x}$ is algebraically equivalent to $1 + x + x^2 + x^3 + x^4 + \cdots$. But as a statement of arithmetic it can be troublesome. For example, it is patently clear that $1 + 3 + 9 + 27 + 81 + \cdots$ cannot equal $\frac{1}{1-3} = -\frac{1}{2}$.

Which topic of mathematics attends to the question: *When can an infinite sum have a meaningful value in arithmetic?*

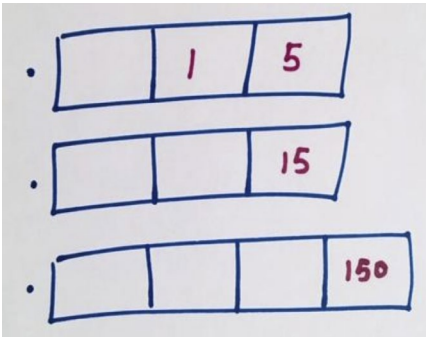
(A) Calculus
(B) Trigonometry
(C) Statistics
(D) Geometry

EXPERIENCE 8: DECIMALS

Question 50: Which is a true reading of the number 0.4 ?

(A) Reading it as “point four.”
(B) Reading it as “four tenths.”

Question 51: When asked to draw a $1 \leftarrow 10$ machine picture of the decimal 0.015 three students submitted the following three diagrams.



The teacher should

(A) Mark all three diagrams as mathematically valid.
(B) Mark just one diagram as mathematically valid.
(C) Mark just two diagrams as mathematically valid.
(D) Mark no diagram as mathematically valid.

Question 52: The decimal 0.0201 is equivalent to which fraction?

(A) $\frac{201}{100}$
(B) $\frac{201}{1000}$
(C) $\frac{201}{10000}$
(D) $\frac{201}{100000}$

Question 53: As a fraction, the decimal 0.0375 equals

(A) $\frac{3}{40}$
(B) $\frac{3}{50}$
(C) $\frac{3}{80}$
(D) $\frac{3}{4}$
(E) $\frac{3}{5}$
(F) $\frac{3}{8}$

Question 54: Scientists will write down the value of a measurement as "12.0 centimeters," for example, rather than just "12 centimeters" because …

(A) They don't understand that 12.0 and 12 are the same numerical value.
(B) They are indicating to the reader the level of precision to which they conducted the measurement.

Question 55: The quantity $0.03 - 0.002$ is equivalent to which of the following? Check all that apply.

(A) $0.0\,|\,3\,|\,-2$
(B) $0.0\,|\,2\,|\,8$
(C) $\frac{7}{25}$
(D) $\frac{7}{250}$

Question 56: The quantity $\frac{1}{0.22+0.08}$ is equivalent to which of the following? Check all that apply.

(A) $\frac{1}{0.3}$
(B) $\frac{1}{0.03}$
(C) $\frac{10}{3}$
(D) $\frac{100}{3}$
(E) $\frac{0.1}{0.3}$
(F) $\frac{0.1}{0.03}$

Question 57: Check all the fractions below that must have infinitely many non-zero digits in their decimal equivalents.

(A) $\frac{1}{12}$
(B) $\frac{2}{12}$
(C) $\frac{3}{12}$
(D) $\frac{4}{12}$
(E) $\frac{5}{12}$
(F) $\frac{6}{12}$

Question 58: Compute $1 \div 2$ in a $1 \leftarrow 5$ machine. That is, find the "decimal" representation of the number one-half in base five. Which of the following is true about that representation?

(A) The representation has only finitely many non-zero digits.
(B) The representation is infinitely long and consists of a single repeating digit.
(C) The representation is infinitely long and consists of two different digits that repeat as a pair.
(D) The representation is infinitely long and there is no repeating pattern to it.

Question 59: Check each expression below that represents an irrational number. (Assume the pattern you see in each of these decimal expressions continues.)

(A) $0.3030303030303...$
(B) $0.3030030003000030000003....$
(C) $0.3132313231323132...$
(D) $13.3132313231323132...$
(E) $13.3030030003000030000003...$

AND FINALLY …

Question 60: Do you pledge to take whatever steps you can—be they small or big—to help students experience at least once some of the joy and wonder that mathematics can offer?

(A) I so pledge.