

## Math and Time: PUZZLE SOLUTIONS

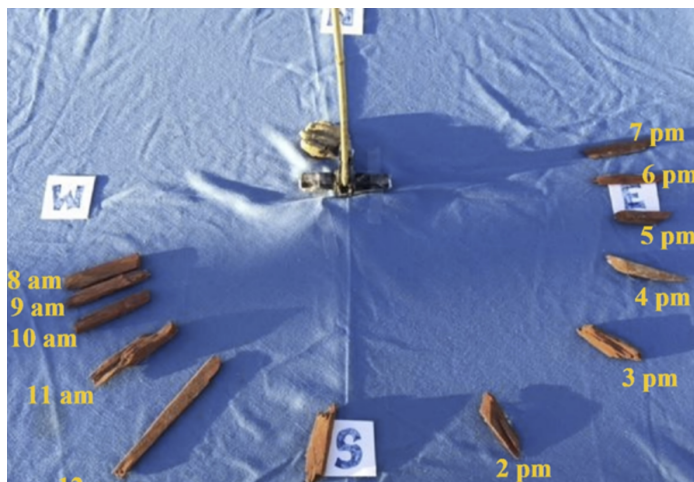
Here are the answers to some of the puzzles from last time.

### puzzle #1

**How many seconds old are you?  
First work out how many days old you  
are, then how many seconds that is.**

#### ANSWER TO PUZZLE 1:

Did you work out your number? Of course, as you read this, your answer keeps changing!



Do you think there is something odd about the photo?

Try sticking a stick in the ground where you live on a sunny day and observing where the shadow lies for different hours throughout the day. For those who live in America, you'll notice something very different!

#### ABOUT THE PHOTO:

I did this activity and took this photo while visiting my family in Australia.

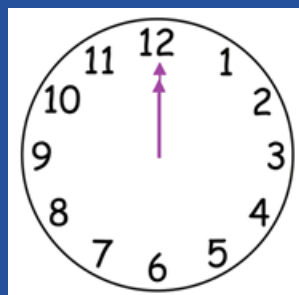
Being in the southern hemisphere, the Sun doesn't move directly overhead, but instead is a little to the north of overhead. This means the shadow of a stick always sits to the south, and as the Sun moves, the shadow moves counterclockwise around the stick.

In America, the Sun is a little to the south of overhead. So the shadow of a stick will always lie on the northern side of the stick and move clockwise around it as the hours pass.

As sundials were extensively first used by peoples in the northern hemisphere, we've come to associate this direction of motion with the passing of time. This is why clocks go clockwise! (But matters would be different if Australian sundials were noticed first!)

### puzzle #2

**The two hands of clock—the minute hand and the hour hand—lie directly on top of one another right at noon.**



**How many times over the next 12 hours will the hands again be perfectly aligned?**

**What times are those?**

## ANSWER TO PUZZLE 2:

After mulling for a little while, I realized that the hands overlap again just after the 1 o'clock hour. (At 1 o'clock the minute hand is back pointing directly upwards, but the hour hand will be pointing at 1. Five minutes later, the minute hand will be pointing at 1, but the hour hand will have moved on a tiny bit. But sometime soon after that, the hands will be aligned.)



So, every one hour, five minutes, and some number of seconds, the hands overlap. And how many 1 hour, five minute, and some number of seconds periods are there over a 12-hour period? There must be 11 over them!

The hands overlap over the course of 12 hours every  $12 \div 11$  hours, that is, every one and one-eleventh of an hour. That's every 1 hour, 5 minutes, and 27-and-three-elevenths seconds. (Translating this into times of days doesn't sound particularly fun, so I am not going to answer the second part of the question—though it is probably not too hard.)

## About the Author: Dr. James Tanton

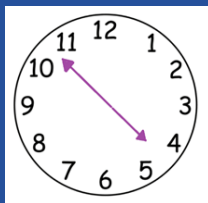


The NMF Weekly is written by mathematician Dr. James Tanton as a resource for friends and fans of the 2021 National Math Festival.

### Learn more:

[globalmathproject.org/nmf-weekly](https://globalmathproject.org/nmf-weekly)  
[nationalmathfestival.org](https://nationalmathfestival.org)

## puzzle #2 BONUS



Are there times of day when the two hands—the minute hand and the hour hand—lie in opposite directions?

## ANSWER TO PUZZLE 2 BONUS:

### WARNING: I am about to do hard work!

There are 60 marks on a clock face, one for each minute. When the minute hand passes through all 60 of them, the hour hand passes through 5. This shows that when the minute hand passes through 1 mark, the hour hand passes through  $1/12$  of one. So after  $T$  minutes, the minute hand passes through  $T$  marks, and the hour hand,  $T/12$  marks.

Is there a time  $T$  where the minute hand is 30 minute marks further than the hour hand? This means we would have

$$T = T/12 + 30.$$

Multiplying through by 12 we get  $12T = T + 360$ , and so  $11T = 360$  and  $T = 360/11 = 32\text{-and-eight-elevenths}$  minutes.

So at time 12:32 and 43-and-seven-elevenths seconds, the hands of the clock are so aligned.

And just like the previous puzzle, this will happen 11 times over the next 12 hours.