



Exploding Dots™ *Teaching Guide*

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Related resources:

- Access the videos of all *Exploding Dots* lessons at:
<http://gdaymath.com/courses/exploding-dots/>
- The *Teaching Guides* for each experience are available [here](#).
- Printable *Handouts* for each experience are available [here](#).

Introduction

Welcome to *Exploding Dots™*, a mathematical journey of delight and wonder!

This journey follows a path through nine experiences, each containing several short lessons. These nine experiences correlate directly with the nine chapters of text that you see here at <http://gdaymath.com/lessons/explodingdots/1-1-welcome/> and with the nine islands you see in Scolab's *Exploding Dots* web application here: <https://gmw.globalmathproject.org>.

This first document gives some general guidance on how to conduct these short lessons with your students, either with full technology available in your classroom or with no, or next-to-no, technology available. Subsequent *Teaching Guide* documents show how to conduct each experience in your class.

Remember, during Global Math Week, we only ask that you conduct a first engagement with *Exploding Dots* with your students for just one class period. There is no rush to complete all that we offer. Keep this first engagement with students uplifting, light, and fun. Everything we offer will remain freely available in perpetuity, so there is plenty of time to continue the story, explore side lessons, and play!

The Core Story

We have identified 15 particular lessons, scattered throughout the first six experiences as essential, “core” lessons for a robust school-mathematics story. Be sure that you, yourself, make it through these 15 core lessons as preparation, even if you don't intend to go this far with your students. (The outline of the *Exploding Dots* lessons appears at the end of this document.)

All age groups – youngsters, high-schoolers, university students, and professionals alike – will start at the first core lesson, “The $2 \leftarrow 1$ Machine.”

High school students and adults will likely move through all, or almost all, of the 15 core lessons in a fast-paced hour or so. They could get to polynomial division and a taste of infinite series in Experience 6.

Young students, within a first hour, might start playing with addition and subtraction algorithms (Experiences 1-4), and intermediate students may make it further, to play with long-division the dots-and-boxes way (Experiences 1-5).

There is no need to labor over ideas and test whether or not students have fully mastered the content within an hour. The goal is to generate excitement and provide students the opportunity to simply experience the mathematics they already know in a stunning new light. At a later date, one can always return to *Exploding Dots* and then take the opportunity to follow the tangential lessons, do all the practice problems, and explore the full story all the way through to the end of the ninth experience at a slower pace.

There is no hurry or pressure to complete the course within a certain time frame.

Even if you only have 40 minutes available with your students and simply start with Experiences 1-2 and just *play* with the machines, the experience will be joyful, exciting, and uplifting.

The goal is to enable students' first encounter with *Exploding Dots* to be wildly fun and intellectually thrilling.

Technology in the Classroom: None vs. Some

James Tanton, the lead of the *Exploding Dots* phenomenon, does not use technology in the classroom when conducting *Exploding Dots* activities with students. He simply uses a white board or a chalk board and makes sure each of his students has something to write on and with.

Each *Teaching Guide* for the Experiences contains teaching notes for the core lessons of the experience and shows how to conduct the experience like James does, with no technology.

James has made videos explaining each of his lessons and some teachers like to show these videos before conducting a class discussion. The *Teaching Guides* include links to the videos.

And, of course, if full technology is available in your classroom, the students can conduct the *Exploding Dots* experience solely through Scolab's self-contained on-line web app here: <https://gmw.globalmathproject.org>. After working through the *Teacher Guides* yourself for the 15 core lessons, you will be well equipped to answer any student questions that arise as they work through the app individually or in pairs.

Materials Needed

Teachers:

No matter the level of technology available in your classroom, as the teacher you will need a large white board or chalk board or other means of displaying examples to students as you write.

Warning: It is not possible to “explode,” – that is, erase – dots written with markers on a flip chart. One can do one’s best and just cross them out, but the diagrams then become messy very quickly and hard to read. If you are using a flip chart, consider having sticky notes that are easy to put on and peel off to use as dots.

Students:

Paper and pencil, or personal-sized white board and marker.

Note: Even though it is not easy to erase dots written with pencil on paper, students tend to cope well with this problem and not be held back by it – once the understanding is in place. But, as a teacher, you need the means to demonstrate the way that dots “explode” by actually erasing them (or removing them if using sticky notes and chart paper).

Students can also construct their own dots and boxes with paper squares, or use dishes for boxes and buttons or grapes for dots, or, with squares drawn with chalk for boxes and stones for dots. See <https://www.youtube.com/watch?v=0chOI7JnYSQ&app=desktop> and <https://www.youtube.com/watch?v=cwicTRuLT4Y> for videos of kids using these tools.

Key to Symbols in These Notes

text Text in black indicates verbiage you might use with students.

text Text in blue indicates teacher guidance comments within the lesson.



The balloon demarks an opportunity to build understanding by allowing students to try for themselves – ask a question, or give a problem to solve.



The exploding dots indicate that this is a key learning moment.

Preparation

1. Allow one to two hours to learn about Exploding Dots for yourself first.

Some people find that watching the videos and glancing through the text on James' personal *Exploding Dots* website <http://gdaymath.com/courses/exploding-dots/> is enough for this. The material there exactly matches the materials in these notes. Others prefer to simply work through the Scolab web app <https://gmw.globalmathproject.org>. It too links to all the videos, contains the written content, and gives the means to interact with *Exploding Dots* machines on screen. See the end of this document for a list of the 15 core lessons.

2. Work through the Teacher Guides to decide how many of the 15 core lessons you wish to conduct with your students.

This will, of course, depend on the age of your students and the number of minutes you have for a first *Exploding Dots* experience you will have with them. See [The Core Story](#) section of this document for suggestions for each age group.

3. Decide how many practice problems you wish for you students to try with each core lesson you choose to do.

The *Teacher Guides* walk through the practice problems James does when giving a joyfully, swift, overview. They are just enough to give students that sense of ownership of ideas and wonder that comes from it.

The student handouts offer more practice content than students will have time to do in one class period. **Do not assign it all!** Let this first encounter with *Exploding Dots* be a joyful first experience, with just enough practice to get a sense of the wonder and delight that lies beneath – **a teaser to entice students to want to return for more.**

4. Create any materials you might wish to provide for your students.

Collate the materials you and your students will need for the classroom experience. Decide, for example, if you would like to prepare laminated sheets of blank “machines” for students to write on with erasable markers. Or handouts of the practice materials for students to try later. Or the “wild explorations” challenge thinking problems for students to mull on after the experience. The *Teacher Guides* provide ideas for these things.

5. Take a moment to relax, and smile, and delight in the mathematical joy that is about to come to you and your students!

Outline of Exploding Dots Lessons

The full set of *Exploding Dots* lessons are outlined below; the 15 core lessons are in bold font. The *Teaching Guide* for each experience goes through James' approach to the core lessons of each experience.

EXPERIENCE 1: THE MACHINES

Welcome

Core Lessons:

- **The $1 \leftarrow 2$ Machine**
- **Other Machines**
- **The $1 \leftarrow 10$ Machine**

Optional Content:

- Wild explorations
- Solutions

EXPERIENCE 2: INSIGHT

Welcome

Core Lessons:

- **Explaining the $1 \leftarrow 2$ Machine**
- **Explaining More Machines**
- **We Speak $1 \leftarrow 10$ Machine**

Optional Content:

- Wild Explorations
- Solutions

EXPERIENCE 3: ADDITION AND MULTIPLICATION

Welcome

Core Lessons:

- **Addition**
- **Multiplication**

Optional Content:

- The Traditional Algorithm
- Multiplication by 10
- Long Multiplication
- Wild Explorations
- Solutions

EXPERIENCE 4: SUBTRACTION

Welcome

Core Lessons:

- **Piles and Holes; Dots and Antidots**
- **Subtraction**

Optional Content:

- The Traditional Algorithm
- Wild Explorations
- Solutions

EXPERIENCE 5: DIVISION

Welcome

Core Lessons:

- **Division**
- **Remainders**

Optional Content:

- Division by 10
- Deep Explanation
- The Traditional Algorithm
- Wild Explorations
- Solutions

EXPERIENCE 6: ALL BASES, ALL AT ONCE - POLYNOMIALS

Welcome

Core Lessons:

- **Division in Any Base**
- **A Problem!**
- **Resolution**

Optional Content:

- Remainders
- The Remainder Theorem
- Multiplying, Adding, and Subtracting Polynomials
- Wild Explorations
- Solutions

EXPERIENCE 7: INFINITE SUMS

Welcome

Optional Content:

- Infinite Sums
- Should we believe infinite sums?

EXPERIENCE 8: DECIMALS

Welcome

Optional Content:

- Decimals
- Adding and Subtracting Decimals
- Multiplying and Dividing Decimals
- Converting Fractions to Decimals
- Irrational Numbers
- Decimals in Other Bases
- Wild Explorations
- Solutions

EXPERIENCE 9: WEIRD AND WILD MACHINES

Welcome

Optional Content:

- Base One-and-a-half?
- Does Order Matter?
- Base Two? Base Three?
- Going Really Wild