GLOBAL MATH PROJECT

*Uplifting Mathematics for All*

# *Exploding Dots™*
# *Teaching Guide*

## Experience 5:

## Division



**Related resources:**

- Access videos of *Exploding Dots™* lessons at: http://gdaymath.com/courses/exploding-dots/
- Be sure to review the *Getting Started* guide, available here.
- Printable student handouts for this experience are available here.

# Experience 5: Division
# Overview

## Student Objectives

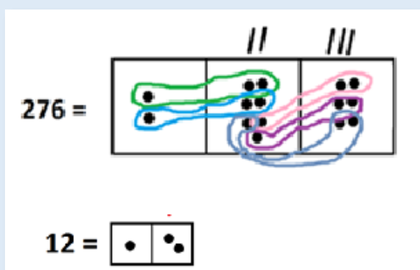
Division can be interpreted as a process of counting: the computation 276 ÷12 is simply asking for the number of groups of 12 one can find in a picture of 276. The visuals of dots-and-boxes make this counting process extraordinarily natural.

## The Experience in a Nutshell

To compute 276 ÷ 12 we could draw a picture of 276 dots on a page and then circle groups of twelve dots. The number of groups of twelve we find is the answer to the division problem.

But this is a highly inefficient way to compute division!

Alternatively, we could draw a picture of 276 in a 1 ← 10 machine and look for groups of twelve in that picture. We readily see that there are two groups at the tens level and three at the ones level, that is, that there are 23 groups in all. We have 276 ÷ 12 = 23.




Remainders are readily identified too. In this example we can see the remainder (R) is 1.


$$277 \div 12 = 23 \quad R\,1$$

## Setting the Scene

View the welcome video from James to set the scene for this experience:
http://gdaymath.com/lessons/explodingdots/5-1-welcome/ [1:43 minutes]

# Division

This is Core Lesson #11, corresponding to Lesson 5.2 on gdaymath.com/courses/exploding-dots/.

James has a video of this lesson here:

http://gdaymath.com/lessons/explodingdots/5-2-division/ [7:38 minutes]

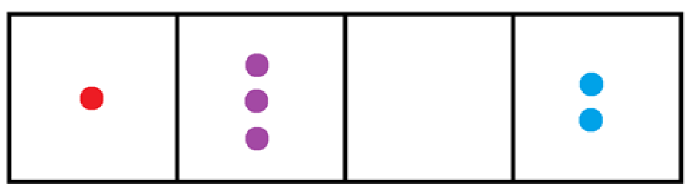
Here is the script James follows when he gives this lesson on a board. Of course, feel free to adapt this wording as suits you best. You will see in the video when and how James draws the diagrams and adds to them.

Addition, subtraction, multiplication. Now it is time for division.

Division is linked with multiplication. In fact, many people think of division as the reverse of multiplication. So let's revisit multiplication for a moment to then see if we can follow it backwards to get to division. We'll start with a straightforward multiplication problem, say, $1302 \times 3$ (with answer $3906$).

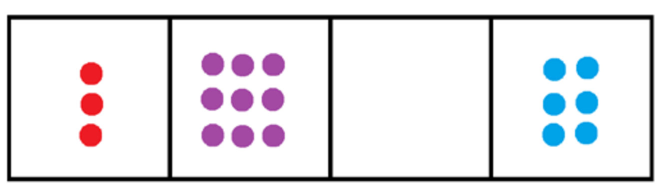
Here's what $1302$ looks like in a $1 \leftarrow 10$ machine. (I've colored the dots for fun.)

**1302 =**

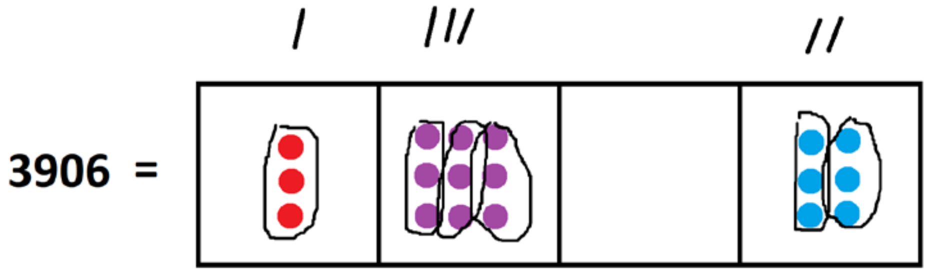


To triple this quantity, we just need to replace each dot in the picture with three dots. We see the answer $3906$.

**1302 x 3 =**



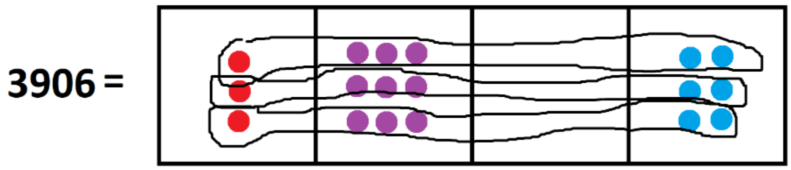
Now suppose I gave you the second picture first and said "Please use it to divide $3906$ by three." Could you use the picture to see triples of dots that must have come from single dots? Yes! Look at the picture this way.



We see the picture as the result of tripling one dot at the thousands level, tripling three dots at the hundreds level, and tripling two dots at the ones level. That is, we see $3906$ as the number $1302$ tripled.

We have just deduced, from the picture, that $3906 \div 3 = 1302$!

Some students might point out that if you look at this picture horizontally you see three copies of 1302 directly.



This picture is neat to see, but we are lucky that there are no explosions to obscure the pattern. In more complicated examples it can be tricky to see this horizontal structure.

So, to divide a number by three, all we need to do is to look for groups of three in the picture of the number. Each group of three corresponds to a dot that must have been tripled. We can just read off the answer to the division problem then by looking at the groups we find!

Of course, we can do the same for any single-digit division problem.

For younger students it might be good to have them practice a single-digit problem on their own.

> *Draw a dots and boxes picture of the number* $426$ *and use it to explain why* $426 \div 2$ *equals* $213$.

If it comes up as a question from students, have them attempt this problem.

> *Try doing* $402 \div 3$ *with just a dots-and-boxes picture. Do you see that unexplosions unlock this problem to reveal the answer* $134$*?*

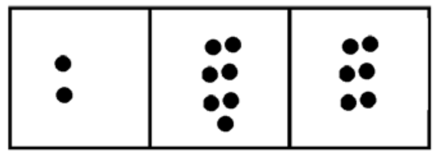
## Multi-Digit Division

Division by single-digit numbers is all well and good. What about division by multi-digit numbers? People usually call that *long division*.

Let's consider the problem $276 \div 12$.

Here is a picture of $276$ in a $1 \leftarrow 10$ machine.

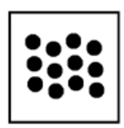
**276 =**



And we are looking for groups of twelve in this picture of $276$. Could this picture be the result of a different picture whose single dots were each replaced by a group of twelve?

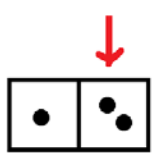
Here's what twelve looks like.

**12 =**



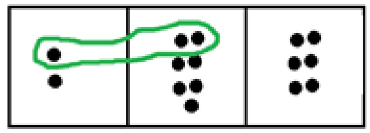
Actually, this is not quite right as there would be an explosion in our $1 \leftarrow 10$ machine. Twelve will look like one dot next to two dots. (But we need to always keep in mind that this really is a picture with all twelve dots residing in the rightmost box.)

**12 =**

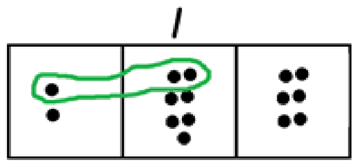


Okay. So we're looking for groups of $12$ in our picture of $276$. Do we see any one-dot-next-to-two-dots in the diagram?

Yes. Here's one.

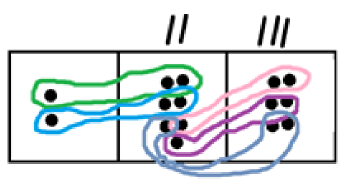
276 = 

Within each loop of $12$ we find, the $12$ dots actually reside in the right part of the loop. So we have found one group of $12$ at the tens level.

276 = 

This point is subtle: the twelve dots of the loop are sitting in the right part of the loop. One might have to remind students a few times as the division process is conducted. It helps to ask: If you were to unexplode some dots just within the loop, which dots could unexplode (and stay in the loop) and in which box would all twelve dots finally be?

And there are more groups of twelve.

276 = 

This shows that our picture of $276$ is actually a picture of $23$ each of whose dots was replaced by $12$. We see $276 = 23 \times 12$ and so $276 \div 12 = 23$.

Alternatively, we can say in our picture of $276$ we've identified two groups of $12$ at the tens level and three $12$s at the ones level, and so there are $23$ groups of $12$ in $276$.

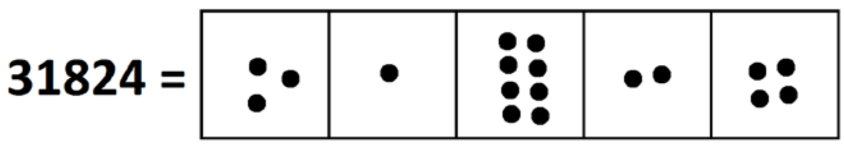
No matter how you wish to interpret matters, the picture reveals all!

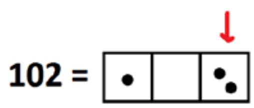
**IMPORTANT: I LEAVE THE PICTURE OF 276 ÷ 12 ON THE BOARD! THIS IS KEY IF YOU ARE GOING TO DO THE SECTION ON POLYNOMIALS IN EXPERIENCE 6.** I am surreptitious about this, making it look as though I just didn't happen to erase that part of the board, even as we go on to more examples and more work.

At this point I might ask students to try one "on their ownsies": 2783 ÷ 23 and then 31824 ÷102 and mention that this second one "has a hiccup." Or, I might just do 31824 ÷ 102 next as part of the lecture. It all depends on what I feel is appropriate for the students.

Let's do another example. Let's compute $31824 \div 102$.

Here's the picture.

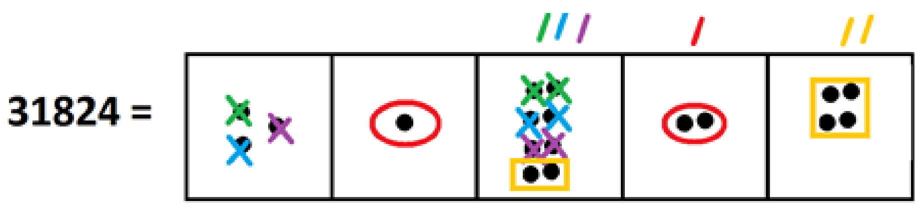



Now we are looking for groups of one dot–no dots–two dots in our picture of $31824$. (And, remember, all $102$ dots are physically sitting in the rightmost position of each set we identify.)

This is the same subtle point as before.

We can spot a number of these groups. (I now find drawing loops messy so I am drawing Xs and circles and boxes instead. Is that okay? Do you also see how I circled a double group in one hit at the very end?)




The answer $312$ to $31824 \div 102$ is now apparent. (The colors show that $31824$ is a picture of $312$ with each dot multiplied by $102$. Or, if you prefer, we've found $312$ groups of $102$ in our picture of $31824$.)

I do usually take the time to explain how the dots-and-boxes method of division aligns with the traditional algorithm. It is tricky for me to write about this here, different countries have different notational systems for long division and slightly different approaches to the algorithm. For example, in the U.S. it has become customary to think of long division as "repeated subtraction," whereas in Serbia and Australia, for example, a more mysterious algorithm is taught.

Watch the video of lesson 5.6 here http://gdaymath.com/lessons/explodingdots/5-6-remainders/ to see the Australian approach and what I usually do for students right at this moment, you can also see the U.S. approach in the text that follows that video in Lesson 5.6 on the site. For your own country's approach, I am afraid to say, it is up to you to figure out the connection to dots-and-boxes (it is likely to be very similar), and to decide if you want to share it with your students at this point.

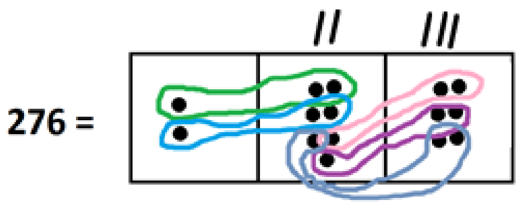
## Remainders

This is Core Lesson #12, corresponding to Lesson 5.4 on gdaymath.com/courses/exploding-dots/.

James has a video of this lesson here:

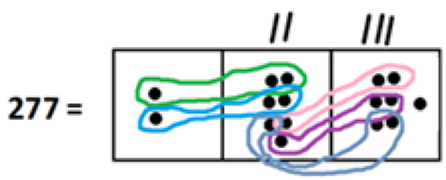
http://gdaymath.com/lessons/explodingdots/5-4-remainders/ [1:59 minutes]

We saw that 276 ÷ 12 equals 23.




Suppose we tried to compute 277 ÷ 12 instead. What picture would we get? How should we interpret the picture?

Well, we'd see the same picture as before except for the appearance of one extra dot, which we fail to include in a group of twelve.




This shows that 277 ÷ 12 equals 23 with a remainder of 1.

You might write this as

$$277 \div 12 = 23\ R\ 1$$

or with some equivalent notation for remainders. (People use different notations for remainders in different countries.)

Or you might be a bit more mathematically precise and say that 277 ÷ 12 equals 23 with one more dot still to be divided by twelve:

$$277 \div 12 = 23 + \frac{1}{12}.$$

(Some people might prefer to think of that single dot as one-twelfth of a group of twelve. All interpretations are good!)

**If you alter the picture of 276 ÷ 12 on the board by adding a dot to make 277 ÷ 12, be sure to casually erase the extra dot and return the picture to 276 ÷ 12. We need this picture for when we move to work on polynomials in Experience 6.**

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## Handout A: *Division and Remainders*

Use the student handout shown below for students who want practice questions from this lesson to mull on later at home. This is NOT homework; it is entirely optional. (See the document “Experience 5: Handouts” for a printable version.)

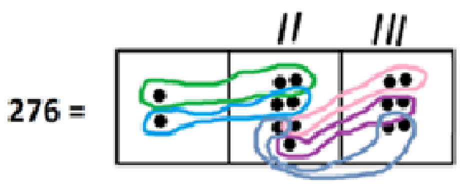
# *Exploding Dots*

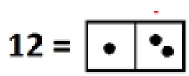
## Experience 5: Division

Access videos of all *Exploding Dots* lessons at: http://gdaymath.com/courses/exploding-dots/

### Handout A: *Division and Remainders*

This picture shows that 276 ÷ 12 equals 23.



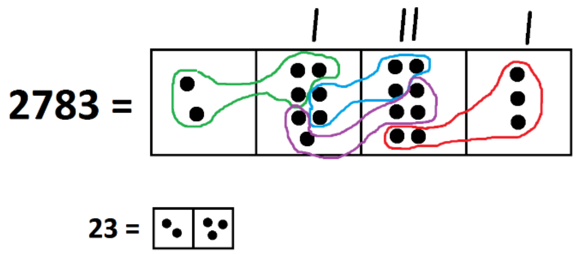




**Here are some practice questions you might, or might not, want to try.**

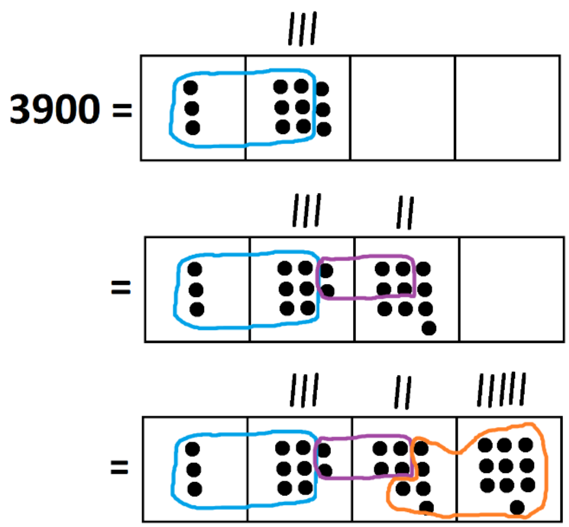
1. Compute 2783 ÷ 23 by the dots-and-boxes approach by hand.
2. Compute 3900 ÷ 12.
3. Compute 46632 ÷ 201.
4. Show that 31533 ÷ 101 equals 312 with a remainder of 21.
5. Compute 2789 ÷ 11.
6. Compute 4366 ÷ 14.
7. Compute 5481 ÷ 131.
8. Compute 61230 ÷ 5.

# Solutions to Handout A

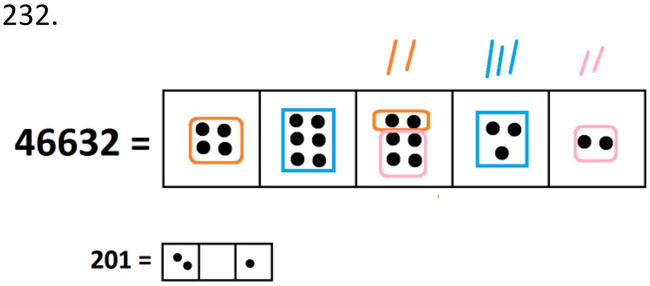
1. $2783 \div 23 = 121$



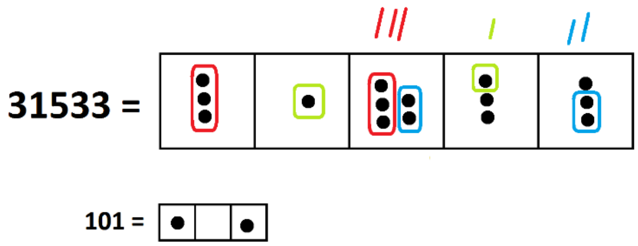

2. $3900 \div 12 = 325$. We need some unexplosions along the way. (And can you see how I am getting efficient with my loop drawing?)



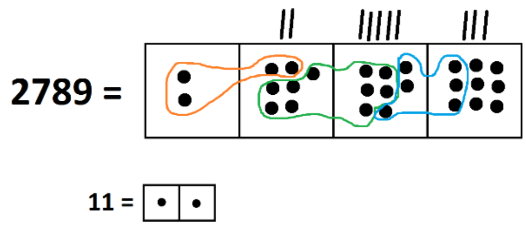

3. $46632 \div 201 = 232$.



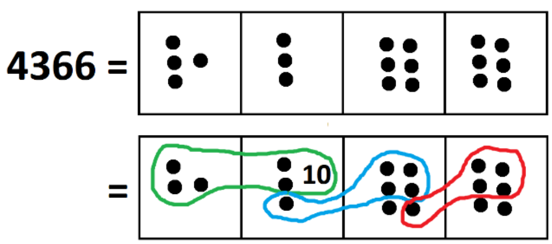
4. $31533 \div 101 = 312$ with a remainder of 21. That is, $31533 \div 101 = 312 + \frac{21}{101}$



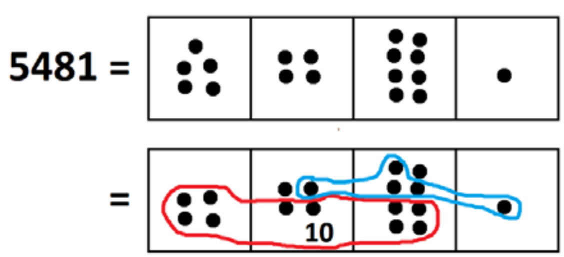

5. We have $2789 \div 11 = 253$ with a remainder of 6. That is, $2789 \div 11 = 253 + \frac{6}{11}$.



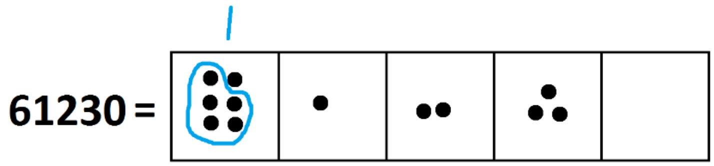

6. $4366 \div 14 = 311 + \frac{12}{14}$.



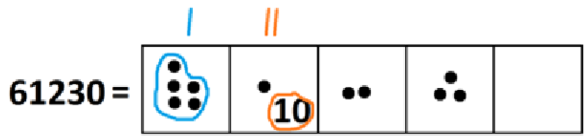

7. $5481 \div 131 = 41 + \frac{110}{131}$.



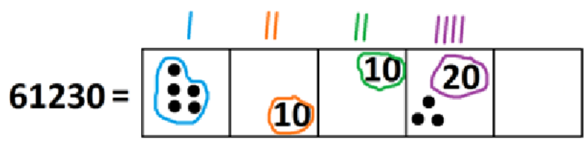
8. We certainly see one group of five right away.

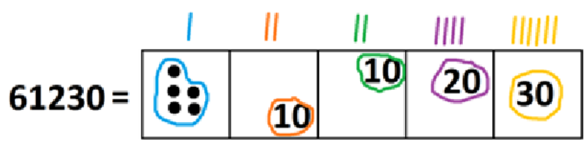



Let's perform some unexplosions. (And let's write numbers rather than draw lots of dots. Drawing dots gets tedious!)








We see $61230 \div 5 = 12246$.

## Handout B: *Wild Explorations*

Use the student handout shown below for students who want some deep-thinking questions from this Experience to mull on later at home. This is NOT homework; it is entirely optional, but this could be a source for student projects. (See the document "Experience 5: Handouts" for a printable version.)

# *Exploding Dots*

## Experience 5: Division

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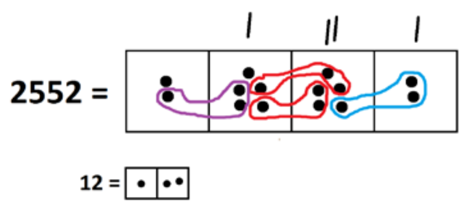
### Handout B: *WILD EXPLORATIONS*

**Here is a "big question" investigation you might want to explore, or just think about. Have fun!**

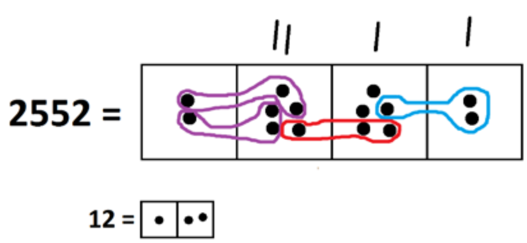
#### *EXPLORATION: LEFT TO RIGHT? RIGHT TO LEFT? ANY ORDER?*

When asked to compute 2552÷12, Kaleb drew this picture, which he got from identifying groups of twelve working right to left.




He said the answer to 2552 ÷ 12 is 121 with a remainder of 1100.

Mabel, on the other hand, identified groups of twelve from left to right in her diagram for the problem.




She concluded that 2552 ÷ 12 is 211 with a remainder of 20. Both Kaleb and Mabel are mathematically correct, but their teacher pointed out that most people would expect an answer with smaller remainders: both $1100$ and $20$ would likely be considered strange remainders for a problem about division by twelve. She also showed Kaleb and Mabel the answer to the problem that is printed in the textbook.

$$2552 \div 12 = 212 \;\; R \; 8$$

How could Kaleb and Mabel each continue work on their diagrams to have this textbook answer appear?