Exploding Dots™

HANDOUTS

Experience 3:

Addition and Multiplication

Exploding Dots

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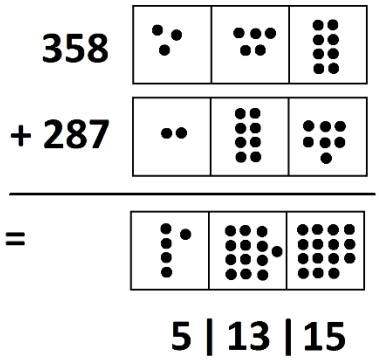
Access videos of all *Exploding Dots* lessons at: http://gdaymath.com/courses/exploding-dots/

Handout A: *Addition*

Here is the *Exploding Dots* way to add 358 and 287.



Explosions then show that this answer is equivalent to 645.

Write down the answers to the following addition problems working left to right and not worrying about what society thinks! Then, do some explosions to translate each answer into something society understands.

148 + 323 =	567 + 271 =	377 + 188 =	582 + 714 =

310462872 + 389107123 =	87263716381 + 18778274824 =

Solutions to Handout A

$148 + 323 = 4 \,|\, 6 \,|\, 11 = 471$

$567 + 271 = 7 \,|\, 13 \,|\, 8 = 838$

$377 + 188 = 4 \,|\, 15 \,|\, 15 = 5 \,|\, 5 \,|\, 15 = 565$

$582 + 714 = 12 \,|\, 9 \,|\, 6 = 1 \,|\, 2 \,|\, 9 \,|\, 6 = 1296$

$310462872 + 389107123 = 6 \,|\, 9 \,|\, 9 \,|\, 5 \,|\, 6 \,|\, 9 \,|\, 9 \,|\, 9 \,|\, 5 = 699569995$

$87263716381 + 18778274824 = 9 \,|\, 15 \,|\, 9 \,|\, 13 \,|\, 11 \,|\, 9 \,|\, 8 \,|\, 10 \,|\, 11 \,|\, 10 \,|\, 5$
$= \ldots = 106041991205$

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Handout B: *Multiplication*

We see that

$$26417 \times 3 = 6\,|\,18\,|\,12\,|\,3\,|\,21$$

2	6	4	1	7	x 3	=	6	18	12	3	21

With explosions, this answer can be rewritten 79251.

Here are some more questions you might or might not choose to ponder.

Compute each of the following: 26417×4, 26417×5, and 26417×9.

Compute 26417×10 and explain why the answer has to be 264170.

(This answer looks like the original number with the digit zero tacked on to its end.)

Extra: Care to compute 26417×11 and 26417×12 too?

(The answer could be, "No! I do not care to do this!)

Solutions to Handout B

We have

$$26417 \times 4 = 8 \,|\, 24 \,|\, 16 \,|\, 4 \,|\, 28 = 10 \,|\, 4 \,|\, 16 \,|\, 4 \,|\, 28 = 1 \,|\, 0 \,|\, 4 \,|\, 16 \,|\, 4 \,|\, 28 = 1 \,|\, 0 \,|\, 5 \,|\, 6 \,|\, 4 \,|\, 28 = 105668$$

$$26417 \times 5 = 10 \,|\, 30 \,|\, 20 \,|\, 5 \,|\, 35 = 10 \,|\, 30 \,|\, 20 \,|\, 8 \,|\, 5 = 10 \,|\, 32 \,|\, 0 \,|\, 8 \,|\, 5 = 13 \,|\, 2 \,|\, 0 \,|\, 8 \,|\, 5 = 132085$$

$$26417 \times 9 = 18 \,|\, 54 \,|\, 36 \,|\, 9 \,|\, 63 = 18 \,|\, 54 \,|\, 36 \,|\, 15 \,|\, 3 = \ldots = 237753$$

$$26417 \times 10 = 20 \,|\, 60 \,|\, 40 \,|\, 10 \,|\, 70 = \cdots = 264170$$

and

$$26417 \times 11 = 22 \,|\, 66 \,|\, 44 \,|\, 11 \,|\, 77 = \ldots = 290587$$

$$26417 \times 12 = 24 \,|\, 72 \,|\, 48 \,|\, 12 \,|\, 84 = \ldots = 317004$$

For a full discussion as to why 26417×10 is 264170 have a look at have a look at the final section of this experience.

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Handout C: *WILD EXPLORATIONS*

Here are some "big question" investigations you might want to explore, or just think about. Have fun!

EXPLORATION 1: THERE IS NOTHING SPECIAL ABOUT BASE TEN FOR ADDITION

Here is an addition problem in a 1 ← 5 machine. (That is, it is a problem in base five.) This is not a 1 ← 10 machine addition.

$$\begin{array}{r} 20413 \\ +\ 13244 \\ \hline \end{array}$$

a) What is the 1 ← 5 machine answer?

b) What number has code 20413 in a 1 ← 5 machine? What number has code 13244 in a 1 ← 5 machine? What is the sum of those two numbers and what is the code for that sum in a 1 ← 5 machine?

[Here are the answers so that you can check your clever thinking.

The sum, as a 1 ← 5 machine problem, is

$$20413 + 13244 = 3\,|\,3\,|\,6\,|\,5\,|\,7 = 3\,|\,4\,|\,1\,|\,5\,|\,7 = 3\,|\,4\,|\,2\,|\,0\,|\,7 = 3\,|\,4\,|\,2\,|\,1\,|\,2 = 34212.$$

In a 1 ← 5 machine, 20413 is two 625's, four 25's, one 5, and three 1's, and so is the number 1358 in base ten; 13244 is the number 1074 in base ten; and 34212 is the number 2432 in base ten. We have just worked out $1358 + 1074 = 2432$.]

EXPLORATION 2: THERE IS NOTHING SPECIAL ABOUT BASE TEN FOR MULTIPLICATION

Let's work with a $1 \leftarrow 3$ machine.

a) Find 111×3 as a base three problem. Also, what are 1202×3 and 2002×3?

Can you explain what you notice?

Comment: For base three, we could write "10" here instead of "3".

Let's now work with a $1 \leftarrow 4$ machine.

b) What is 133×4 as a base four problem? What is 2011×4? What is 22×4?

Can you explain what you notice?

Comment: For base four, we could write "10" here instead of "4".

In general, if we are working with a $1 \leftarrow b$ machine, can you explain why multiplying a number in base b by b returns the original number with a zero tacked on to its right?