



Uplifting Mathematics for All

Exploding Dots™

HANDOUTS

Experience 2: Insight

<u>Handout A: Explaining the Machines</u>	2
<u>Solutions to Handout A</u>	4
<u>Handout B: Wild Explorations</u>	5

Exploding Dots

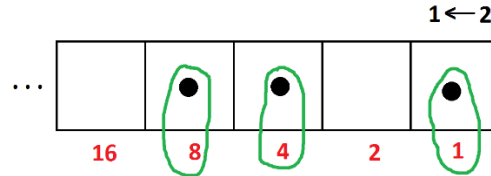
Experience 2: Insight

Access videos of all *Exploding Dots* lessons at: <http://gdaymath.com/courses/exploding-dots/>

Handout A: Explaining the Machines

In a $1 \leftarrow 2$ machine a pair of dots in any one box is equivalent to a single dot, one box to their left. As dots in the rightmost box are worth 1, dots in subsequent boxes are thus worth 2, 4, 8, 16, and so on.

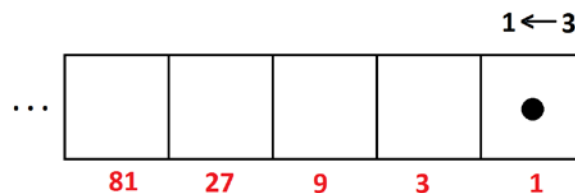
We can see that the $1 \leftarrow 2$ machine code 1101 for the number thirteen, for instance, is correct: thirteen is one 8, one 4, and one 1.



Here are some questions you might, or might not, want to try:

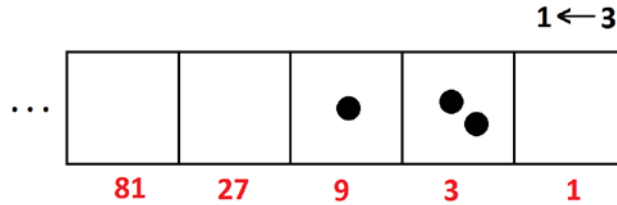
1. What number has $1 \leftarrow 2$ machine code 100101?
2. What is the $1 \leftarrow 2$ machine code for the number two hundred?

In a $1 \leftarrow 3$ machine, three dots in any one box are equivalent to one dot one place to the left. (And each dot in the rightmost box is again worth 1.) We get the dot values in this machine by noting that three 1's is 3, and three 3's is 9, and three 9's is 27, and so on.



3.

- a) What is the value of a dot in the next box to the left after the ones shown?
- b) The $1 \leftarrow 3$ machine code for fifteen is 120. We see that this is correct as one 9 and two 3's does indeed make fifteen.



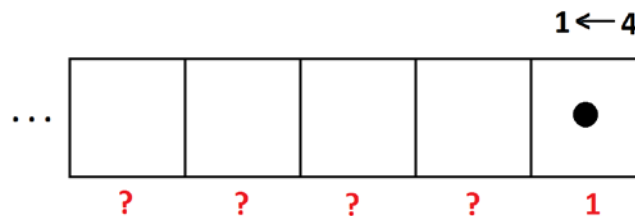
Could we also say that the $1 \leftarrow 3$ code for fifteen is 0120? That is, is it okay to put zeros in the front of these codes? What about zeros at the ends of codes? Are they optional?

Is it okay to leave off the last zero of the code 120 for fifteen and just write instead 12?

- c) What number has $1 \leftarrow 3$ machine code 21002?
- d) What is the $1 \leftarrow 3$ machine code for two hundred?

4.

- a) In the $1 \leftarrow 4$ system four dots in any one box are equivalent to one dot, one place to their left. What is the value of a dot in each box?



- b) What is the $1 \leftarrow 4$ machine code for twenty-nine?
- c) What number has 132 as its $1 \leftarrow 4$ machine code?
5. I happen to know that *Venutians* have six fingers on each of two hands. What base do you think they might use in their society?

Solutions to Handout A

1. Thirty-seven. It's a 32 and a 4 and a 1.

2. 11001000

3.

a) Each dot in the next box to the left is worth three 81's, that's 243.

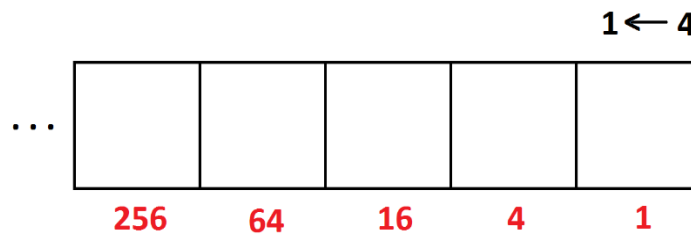
b) Yes it is okay to insert a zero at the front of the code. This would say that there are no 27's, which is absolutely correct. Deleting the end zero at the right, however, is problematic. 120 is the code for fifteen (one 9 and two 3's) but 12 is the code for five (one 3 and two 1's).

c) One hundred and ninety one. (Two 81's, one 27, and two 1's.)

d) 21102

4.

a) For a $1 \leftarrow 4$ machine, boxes have the following values:



b) The number twenty-nine has code 131 in a $1 \leftarrow 4$ machine.

c) Thirty. (This is one more than the code for twenty-nine!)

5. Might Venutians use base twelve? This means they will need twelve different symbols for writing numbers.

By the way, have you noticed that we use ten different symbols – 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0 – which we call *digits*. (We call our fingers *digits* too!)

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Handout B: *WILD EXPLORATIONS*

Here are some “big question” investigations you might want to explore, or just think about. Have fun!

EXPLORATION 1: CAN MACHINES “GO THE OTHER WAY”?

Jay decides to play with a machine that follows a $1 \leftarrow 1$ rule. He puts one dot into the right-most box. What happens? Do assume there are infinitely many boxes to the left.



Suggi plays with a machine following the rule $2 \leftarrow 1$. She puts one dot into the right-most box. What happens for her?

Do you think these machines are interesting? Is there much to study about them?

EXPLORATION 2: CAN WE PLAY WITH WEIRD MACHINES?

Poindexter decides to play with a machine that follows the rule $2 \leftarrow 3$.

- a) Describe what happens when there are three dots in a box.
- b) Work out the $2 \leftarrow 3$ machine codes for the numbers 1 up to 30. Any patterns?
- c) The code for ten in this machine turns out to be 2101. Look at your code for twenty. Can you see it as the answer to “ten plus ten”? Does your code for thirty look like the answer to “ten plus ten plus ten”?

Comment: We’ll explore this weird $2 \leftarrow 3$ machine in Experience 9. It is mighty weird!