Experience 1:
The Machines

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Handout A: The 1 ← 2 Machine

Here’s the rule for a 1 ← 2 machine:

Whenever there are two dots in a box they explode and disappear – KAPOW! – to be replaced by one dot, one box to the left.

And here are some questions you might, or might not, want to try:

1. a) What is the 1 ← 2 machine code for the number thirteen?

   (It turns out to be 1101. Can you get that answer?)

   b) What is the code for fifty in this machine? (Whoa!)

2. Could a number ever have code 100211 in a 1 ← 2 machine, assuming we always choose to explode dots if we can?

3. **CHALLENGE:** Which number has code 10011 in a 1 ← 2 machine?
Solutions to Handout A

1.  
   a) Here’s how the code 1101 appears from thirteen dots.

   
   \[
   \begin{array}{cccccc}
   \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
   \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
   \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
   \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
   \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
   \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
   \end{array}
   \]

   b) The number fifty has code 110010.

2.  
   Assuming we want to make the agreement that we’ll always choose to explode dots if we can, then the code 100211 is not complete: the two dots in the third-to-last box can explode to give a final code of 101011.

3.  
   This is the code for the number nineteen. (In the next experience, we’ll discover a swift way to see this.)
Exploding Dots
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**Handout B: Other Machines**

Here are some more questions you might, or might not, choose to ponder.

1. a) Show that the code for four in a $1 \leftarrow 3$ machine is 11.
   b) Show that the code for thirteen in a $1 \leftarrow 3$ machine is 111.
   c) Show that the code for twenty in a $1 \leftarrow 3$ machine is 202.

2. Could a number have code 2041 in a $1 \leftarrow 3$ machine? If so, would the code be “stable”?

3. Which number has code 1022 in a $1 \leftarrow 3$ machine?

We can keep going!

4. What do you think rule is for a $1 \leftarrow 4$ machine?
   What is the $1 \leftarrow 4$ code for the number thirteen?

5. What is the $1 \leftarrow 5$ code for the number thirteen?

6. What is the $1 \leftarrow 9$ code for the number thirteen?

7. What is the $1 \leftarrow 5$ code for the number twelve?

8. What is the $1 \leftarrow 9$ code for the number twenty?

9. a) What is the $1 \leftarrow 10$ code for the number thirteen?
   b) What is the $1 \leftarrow 10$ code for the number thirty-seven?
   c) What is the $1 \leftarrow 10$ code for the number 5846?
Solutions to Handout B

1. a) Do it! b) Do this one too! c) You’re on a roll. Do this third one as well!

2. Again, if we agree to do all the explosions we can, then this code is not complete: three of the dots in the second-to-last box can explode to give 2111.

3. The number thirty-five has this code.

4. “Four dots in any one box explode and are replaced by one dot one place to the left.” The number thirteen has code 31 in a 1 ← 4 machine.

5. 23

6. 14

7. 22

8. 22 (Same code as the previous answer – but, of course, the interpretation of the code is different.)

9. a) 13 b) 37 c) 5846 (These are the codes we use for numbers in everyday life!)
Exploding Dots

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Handout C: WILD EXPLORATIONS

Here are some “big question” investigations you might want to explore, or just think about. All will become clear as the story unfolds in further chapters, but it could be fun to mull on these ideas now.

EXPLORATION 1: WHAT ARE THESE MACHINES DOING?

Can you figure out what these machines are actually doing?

Why is the code for two hundred and seventy-three in a $1 \leftarrow 10$ machine, “273”? Are all the codes for numbers in a $1 \leftarrow 10$ sure to be identical to how we normally write numbers?

If you can answer that question, can you then also make sense of all the codes for a $1 \leftarrow 2$ machine? What does the code 1101 for the number thirteen mean?

Comment: Experience 2 answers these questions.

EXPLORATION 2: DOES THE ORDER IN WHICH ONE EXPLODES DOTS SEEM TO MATTER?

Put nineteen dots into the rightmost box of a $1 \leftarrow 2$ machine and explode pairs of dots in a haphazard manner: explode a few pairs in the rightmost box, and then some in the second box, and then a few more in the rightmost box, and then some in the second box again, and so on. Do it again, this time changing the order in which you do explosions. And then again!

Does the same final code of 10011 appear each, and every time?